Review of CMOS amplifiers

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Basic CMOS amplifiers

Common Emitter

Common Base

Common Collector

Emitter Degeneration

Common Source

Common Gate

Common Drain

Source Degeneration
Common-source amplifier

\[ V_o = V_{DD} - I_d \cdot R_L = V_{DD} - \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_i - V_{tn})^2 \cdot R_L \]

- DC voltage \( V_i \) is chosen to bias M1 so that M1 is in active (saturation) region and its drain voltage is near the midpoint of the output swing \( (V_O \approx V_{DD}/2) \).
Common-source amplifier – small-signal analysis

\[ A_v(0) = -g_m(r_o \parallel R_L) = -g_m R'_L \quad R_i = \infty \quad R_o = r_o \parallel R_L = R'_L \]

- Note that for \( R_L \to \infty \), \( R'_L \to r_o \), and

\[ |A_v(0)|_{max} = g_m r_o = \frac{I_D}{V_{ov}/2} \times \frac{L_{eff}}{I_D} \left| \frac{\partial L_{eff}}{\partial V_{DS}} \right|^{-1} = \frac{2L_{eff}}{V_{ov}} \left| \frac{\partial L_{eff}}{\partial V_{DS}} \right|^{-1} \]
Common-source amplifier – small-signal analysis

\[ v_s = v_i \quad R_1 = R_S \quad R_2 = r_o \parallel R_L \quad C_1 = C_{gs} \quad C_2 = C_L \quad C_f = C_{gd} \]
Miller approximation

\[ v_o = ( -g_m v_1 + i_f ) \left( R_2 \parallel \frac{1}{sC_2} \right) \quad i_f = (v_1 - v_o)sC_f \]

If \( R_2 - C_2 \) is a non-dominant pole, then, at the frequencies of interest

\[ v_o \approx -g_m R_2 v_1 \quad i_f \approx (v_1 + g_m R_2 v_1)sC_f \quad \Rightarrow \quad \frac{i_f}{v_i} = s(1 + g_m R_2)C_f = sC_M \]

\[ C_M = (1 + g_m R_2) \cdot C_f = (1 + a_{v_0}) \cdot C_f = \text{Miller Capacitance} \]
Miller approximation

\[ C_t = C_1 + C_M = C_1 + (1 + g_m R_2) C_f \]

\[ A_v(s) = \frac{v_o}{v_s} = A_v(0) \frac{1}{(1 - s/p_1)(1 - s/p_2)} \]

\[ A_v(0) = -g_m R_2 \quad p_1 = \frac{1}{R_1 C_t} \quad p_2 = \frac{1}{R_2 C_2} \]
MOSFET’s Transition frequency

\[ i_o \approx -g_m \times v_1 = -g_m \times i_1 \frac{R_{1s}}{1 + R_{1s}(C_1 + C_f)s} \]

Short-Circuit Current Gain = \( \beta(s) = -\frac{i_o}{i_i} = \frac{g_m R_{1s}}{1 + R_{1s}(C_1 + C_f)s} \)

Transition Frequency = \( \omega_T \approx \frac{g_m}{C_1 + C_f} \)

\[ R_{1s} = \infty \quad C_1 = C_{gs} = \frac{2}{3} C_{ox} WL \quad C_f = C_{gd} \]

\[ \omega_T = 2\pi f_T = \frac{g_m}{C_{gs} + C_{gd}} \]
MOSFET’s Transition frequency

To calculate intrinsic device speed, let $\omega_T \approx g_m/C_{gs}$.

- For square-law device,

  $$g_m = \mu C_{ox} \frac{W}{L} V_{ov} \quad \Rightarrow \quad \omega_T = \frac{3}{2} \cdot \frac{\mu}{L^2} \cdot V_{ov}$$

- For device with carrier velocity saturation,

  $$g_m = W C_{ox} V_{scI} \quad \Rightarrow \quad \omega_T = \frac{3}{2} \cdot \frac{V_{scI}}{L}$$
MOSFET’s Transition frequency

For MOSTs in the weak inversion region,

\[ \omega_T = \frac{g_m}{C_{gb}} \]

\[ g_m = \frac{I_D}{U_T C_{ox} + C_{depl}} \]

\[ C_{gb} = W L \times \frac{C_{ox} C_{depl}}{C_{ox} + C_{depl}} \]

\[ \omega_T = \frac{I_D}{U_T} \cdot \frac{1}{W L C_{depl}} = \frac{I_t}{U_T} \cdot \frac{1}{C_{depl}} \cdot \frac{1}{L^2} \cdot \frac{I_D}{I_M} \]

- \( I_M = I_t \cdot W/L \) is the maximum \( I_D \) for device in weak inversion.

Since \( I_t \propto D_n \) and \( D_n = \mu U_T \), we have

\[ \omega_T \approx \frac{D_n}{L^2} \cdot \frac{I_D}{I_M} \approx \frac{\mu}{L^2} \cdot U_T \cdot \frac{I_D}{I_M} \]
Common-source amplifier with source degeneration
Common-source amplifier with source degeneration

To find $C_{g_{seq}}$ and $g_{meq}$, let $v_o = 0$, then

$$(g_m + sC_{gs})(v_i - v_s) = (G_S + g_{mb} + g_o)v_s$$

At frequencies where $\omega \ll \omega_T = g_m/C_{gs}$,

$$\frac{v_s}{v_i} = \frac{g_{meq}}{g_{m} + g_{mb} + G_S + g_o + sC_{gs}} = \frac{g_m}{g_{m} + g_{mb} + G_S + g_o}$$

$$g_{meq} = \frac{-i_o}{v_i} = g_m \left( 1 - \frac{v_s}{v_i} \right) = \left( g_{mb} + g_o \right) \frac{v_s}{v_i} = \frac{g_m G_S}{g_{m} + g_{mb} + G_S + g_o} = \frac{g_m}{1 + (g_m + g_{mb})R_S + \frac{R_S}{r_o}}$$

$$\frac{i_i}{v_i} = sC_{gs} \left( 1 - \frac{v_s}{v_i} \right) = sC_{gs} \cdot \frac{1 + g_{mb}R_S + \frac{R_S}{r_o}}{1 + (g_m + g_{mb})R_S + \frac{R_S}{r_o}}$$
Common-source amplifier with source degeneration

- If $r_o \gg R_S$, 

$$g_{meq} \approx \frac{g_m}{1 + (g_m + g_{mb})R_S} = \frac{g_m}{1 + (1 + \chi)g_mR_S}$$

$$C_{gseq} = C_{gs} \cdot \frac{1 + g_{mb}R_S}{1 + (g_m + g_{mb})R_S} = C_{gs} \cdot \frac{1 + \chi g_mR_S}{1 + (1 + \chi)g_mR_S}$$

To find $r_{oeq}$, let $v_i = 0$, then

$$(g_m + g_{mb} + sC_{gs} + G_S)v_s = g_o(v_o - v_s)$$

$$\frac{v_s}{v_o} = \frac{g_o}{g_m + g_{mb} + G_s + g_o + sC_{gs}} \approx \frac{g_o}{g_m + g_{mb} + G_s + g_o}$$

$$g_{oeq} = \frac{i_o}{v_o} = g_o \left(1 - \frac{v_e}{v_o}\right) - (g_m + g_{mb})\frac{v_e}{v_o} = \frac{g_o G_S}{g_m + g_{mb} + G_S + g_o}$$

$$r_{oeq} = R_S + r_o[1 + (g_m + g_{mb})R_S]$$

- $r_{oeq}$ can be made arbitrarily large by increasing $R_S$. 

Mixed-signal ICs Design, A. Thanachayanont 2010
Common-gate amplifier

\[ g'_m = g_m + g_{mb} \quad C'_L = C_t + C_{gd} = C_L + C_{db} + C_{gd} \quad C_{in} = C_{gs} + C'_{sb} \]

The nodal equations are

\[ i_{in} = (g'_m + sC_{in})v_{in} - g_o(v_o - v_{in}) \quad g'_m v_{in} = (G_L + sC'_L)v_o + g_o(v_o - v_{in}) \]
Common-gate amplifier

If the $g_o(v_o - v_{in})$ terms are neglected, then

$$Transimpedance = Z_t(s) = \frac{v_o}{i_{in}} = \frac{R_L}{(1 - s/p_1)(1 - s/p_2)}$$

$$p_1 = -\frac{g_m'}{C_{in}} = -\frac{g_m'}{C_{gs} + C_{sv}}$$

$$p_2 = -\frac{1}{R_L C'_L}$$

$$Input\ Impedance = Z_{in}(s) = \frac{v_{in}(s)}{i_{in}(s)} = \frac{1/g_m'}{1 - s/p_1}$$

$$Current\ Gain = \frac{i_o(s)}{i_{in}(s)} = \frac{g_m' v_{in}}{i_{in}} = g_m' Z_{in}(s) = \frac{1}{1 - s/p_1}$$

- Note that $p_1 \approx \omega_T = g_m/(C_{gs} + C_{gd})$. 
Common-gate amplifier

If \( g_o \) is considered,

\[
\text{Voltage Gain} = A_v(s) = \frac{v_o}{v_{in}} = \frac{g'_m + g_o}{g_o + G_L + sC'_L}
\]

\[
\text{Input Impedance} = Y_{in}(s) = \frac{i_{in}}{v_{in}} = g'_m + sC_{in} - g_o(A_v - 1)
\]

\[
Z_t(s) = \frac{v_o}{i_{in}} = \frac{A_v(s)}{Y_{in}(s)}
\]

\bullet At low frequencies where \( \omega \to 0 \), assuming \( g'_m \gg g_o \),

\[
A_v = \frac{g'_m + g_o}{g_o + G_L} \approx \frac{g'_m}{g_o + G_L}
\]

\[
Y_{in} = g'_m - \frac{g'_m}{1 + \frac{G_L}{g_o}} + g_o \approx \frac{g'_m}{1 + \frac{R_L}{r_o}}
\]

\[
Z_t = \frac{A_v}{Y_{in}} \approx R_L
\]
Common-drain amplifier

\[ C'_S = C_S + C_{gd1} \quad C'_L = C_L + C'_{sb1} + C_{db2} + C_{gd2} \quad G'_L = g_{o1} + g_{o2} + g_{mb1} = \frac{1}{R'_L} \]
Common-drain amplifier

Summing the currents at the output node, we have

$$(g_{m1} + sC_{gs1})(v_g - v_o) - v_o(sC'_L + G'_L) = 0$$

The voltage gain from gate to output is

$$A_{vg}(s) = \frac{v_o(s)}{v_g(s)} = \frac{g_{m1} + sC_{gs1}}{g_{m1} + G'_L + s(C_{gs1} + C'_L)} = A_{vg}(0) \frac{1 - s/z_1}{1 - s/p_1}$$

$$A_{vg}(0) = \frac{g_{m1}}{g_{m1} + G'_L} = \frac{g_{m1}}{g_{m1} + g_{mb1} + g_{o1} + g_{o2}}$$

$$A_{vg}(\infty) = \frac{C_{gs1}}{C_{gs1} + C'_L}$$

$$z_1 = -\frac{g_{m1}}{C_{gs1}} \approx -\omega_T$$

$$p_1 = -\frac{g_{m1} + G'_L}{C_{gs1} + C'_L}$$
Common-drain amplifier

For most practical cases

\[ g_{o1} + g_{o2} \ll g_{m1} + g_{mb1} = g_{m1}(1 + \chi) \]

\[ A_{vg}(0) \approx \frac{g_{m1}}{g_{m1} + g_{mb1}} \]

\[ \approx \frac{1}{1 + \chi_1} \]

\[ p_1 \approx -\frac{g_{m1}(1 + \chi_1)}{C_{gs1} + C'_L} \]

\[ \approx (1 + \chi_1) \left( \frac{1}{1 + \frac{C'_L}{C_{gs1}}} \right) z_1 \]

\[ |p1| > |z1| \]

\[ (C'_L = 0) \]

\[ |p1| = |z1| \]

\[ A_{vg}(0) \]

\[ |p1| < |z1| \]

\[ A_{vg}(0) \]
Common-drain amplifier

The input admittance looking into the gate is

\[ Y_g(s) = \frac{i_g}{V_g} = sC_{gs1}[1 - A_{vg}(s)] = \frac{sC_{gs1}(G_L' + sC_L')}{g_{m1} + G_L' + s(C_{gs1} + C_L')} \]

Define the capacitance looking into the gate as

\[ Y_g(s) = sC_g(s) \]

\[ C_g(j\omega) = C_{gs1}[1 - A_{vg}(j\omega)] \]
\[ C_g(0) = C_{gs1}[1 - A_{vg}(0)] \]
\[ C_g(\infty) = C_{gs1}[1 - A_{vg}(\infty)] = \frac{C_{gs1}C_L'}{C_{gs1} + C_L'} \]
Common-drain amplifier

\[ i_o = -(g_{m1} + sC_{gs1})(v_g - v_o) \]

\[ G_s v_g + sC_{gs1}(v_g - v_o) = 0 \]
The output admittance is

\[ Y_o(s) = \frac{1}{Z_o(s)} \equiv \frac{i_o}{v_o} = \frac{G_S(g_{m1} + sC_{gs1})}{G_S + sC_{gs1}} = G_S + \frac{G_S(g_{m1} - G_S)}{G_S + sC_{gs1}} = G_S + \frac{1}{\frac{1}{g_{m1} - G_S} + \frac{sC_{gs1}R}{g_{m1} - G_S}} \]

- Note that

\[ Z_o(0) = \frac{1}{g_{m1}} \quad Z_o(\infty) = R_S \]

- The equivalent circuit is

\[ R_1 = \frac{1}{g_{m1} - G_S} \quad R_2 = R_S \quad L = \frac{R_SC_{gs1}}{g_{m1} - G_S} \]
Dominant pole approximation

The response of an amplifier has the form of

\[
A(s) = A(0) \frac{N(s)}{D(s)} = A(0) \frac{1 + a_1 s + a_2 s^2 + \cdots + a_m s^m}{1 + b_1 s + b_2 s^2 + \cdots + b_n s^n} \approx \frac{A(0)}{\left(1 - \frac{s}{\rho_1}\right) \left(1 - \frac{s}{\rho_2}\right) \cdots \left(1 - \frac{s}{\rho_n}\right)}
\]

If \(|\rho_1| \ll |\rho_2|, |\rho_3|, \ldots, |\rho_n|\), then \(\rho_1\) is a dominant pole. We have

\[
b_1 = -\frac{1}{\rho_1} - \frac{1}{\rho_2} - \cdots - \frac{1}{\rho_n} \approx -\frac{1}{\rho_1} = \left|\frac{1}{\rho_1}\right|
\]

\[
|A(j \omega)| = \frac{A(0)}{\sqrt{\left[1 + \left(\frac{\omega}{\rho_1}\right)^2\right] \left[1 + \left(\frac{\omega}{\rho_2}\right)^2\right] \cdots \left[1 + \left(\frac{\omega}{\rho_n}\right)^2\right]}} \approx \frac{A(0)}{\sqrt{1 + \left(\frac{\omega}{\rho_1}\right)^2}}
\]

\(-3\) dB Bandwidth = \(\omega_{-3\text{dB}} \approx |\rho_1| \approx \frac{1}{b_1}\)
Zero-value time constants

- \( \eta \) is a linear active network without energy storage.

- The \( b_1 \) in the denominator of the system function can be expressed as

\[
b_1 = \sum T_0 = R_{10}C_1 + R_{20}C_2 + R_{30}C_3 + \cdots
\]

\( R_{i0} \) is the driving point resistance seen by \( C_i \) with all capacitors equal to zero.
Zero-value time constants

To determine $R_{f0}$, replace $C_f$ with a current source $i_f$, then

$$v_1 = i_f R_1 \quad v_o = -(i_f + g_m v_1) R_2$$

$$R_{f0} = \frac{v_1 - v_o}{i_f} = R_1 + R_2 + g_m R_1 R_2 = R_1 \left(1 + g_m R_2 + \frac{R_2}{R_1}\right)$$

We have

$$b_1 = \sum T_0 = R_1 C_1 + R_2 C_2 + (R_1 + R_2 + g_m R_1 R_2) C_f$$
Cascode amplifiers

\[ g'_{m2} = g_{m2} + g_{mb2} \quad C_x = C_{db1} + C'_{sb2} + C_{gs2} \quad C'_L = C_L + C_{db2} + C_{gd2} \]
Cascode amplifiers

The output impedance looking into M2’s drain is

\[ R_{ot2} = r_{o1} + (g'_{m2}r_{o1} + 1)r_{o2} \approx g'_{m2}r_{o1}r_{o2} \]

The input admittance looking into M2’s source is

\[ G_{in2} \approx \frac{g'_{m2}}{g_{o2}/G_L + 1} \]

The overall voltage gain is

\[ A_v = \frac{v_o}{v_{in}} \approx -\frac{g_{m1}}{g_{o1} + G_{in2}} \times \frac{g'_{m2}}{g_{o2} + G_L} = g_{m1} \times \frac{g'_{m2}r_{o1}r_{o2}R_L}{r_{o2} + g'_{m2}r_{o1}r_{o2} + R_L} \approx g_{m1} \times (R_{ot2} \parallel R_L) \]

- Let \( g_m = g_{m1} = g'_{m2} \), \( r_o = r_{o1} = r_{o2} \), and \( g_m \gg g_o \). If \( R_L = R_{ot2} = g_m^2r_o \), then

\[ G_{in2} \approx \frac{g_m}{g_oR_L + 1} \approx \frac{g_m}{g_m^2r_o + 1} \approx g_o \quad A_v \approx -\frac{g_m}{2g_o} \frac{g_m}{g_o + G_L} \approx -\frac{1}{2} \left( \frac{g_m}{g_o} \right)^2 \]
Cascode amplifiers

Using the zero-value time constant method, we have

\[ R_{gs10} = R_S \quad R_{gd10} = R_S + R_{x0} + g_m R_S R_{x0} \]

\[ R_{x0} = r_{o1} \parallel R_{in2} \approx r_{o1} \parallel \left(\frac{1}{g'_m} \frac{g_{o2}}{G_L} + 1\right) \quad R_{L0} = R_L \parallel R_{ot2} \approx R_L \parallel (g'_m r_{o1} r_{o2}) \]

\[ \sum T_0 = R_{gs10} C_{gs1} + R_{gd10} C_{gd1} + R_{x0} C_x + R_{L0} C_L \quad \omega_{-3dB} = 1 / \left(\sum T_0\right) \]

- Let \( g_m = g_{m1} = g'_m \), \( r_o = r_{o1} = r_{o2} \), \( g_m \gg g_o \). If \( R_L = R_{ot2} = g_m r_o^2 \) and \( R_S = r_o \), then

\[ R_{in2} \approx r_o \quad R_{x0} = \frac{r_o}{2} \quad R_{L0} \approx \frac{g_m r_o^2}{2} \quad R_{gd10} \approx R_S + \frac{r_o}{2} + \frac{g_m r_o R_S}{2} \approx \frac{g_m r_o^2}{2} \]

\[ \Rightarrow \quad \sum T_0 = R_S C_{gs1} + \frac{g_m r_o^2}{2} C_{gd1} + \frac{r_o}{2} C_x + \frac{g_m r_o^2}{2} C_L \]

- \( R_{L0} C_L \) usually is the dominant term, unless \( R_S \) is very large.
Cascode amplifiers

Let \( R_L = R_{ot2} = g'_m r_{o1} r_{o2} \), then

\[
G_{in2} \approx \frac{g'_m}{g_{o2} R_L + 1} \approx \frac{g'_m}{g'_m r_{o1} + 1} \approx g_{o1}
\]

The dc gain is

\[
A_v(0) \approx -\frac{g_{m1}}{g_{o1} + G_{in2}} \times \frac{g'_m}{g_{o2} + G_L} \approx -\frac{1}{2} \cdot \frac{g_{m1}}{g_{o1}} \cdot \frac{g'_m}{g_{o2}}
\]

The dominant pole is

\[
p_1 = -\frac{1}{R_{L0} C_L} \approx -\frac{2}{g'_m r_{o1} r_{o2} C_L}
\]

At frequencies where \(|p_1| \ll \omega \ll |p_2|\),

\[
A_v(s) = \frac{A_v(0)}{1 - \frac{s}{p_1}} \approx \frac{A_v(0)}{-\frac{s}{p_1}} \approx -\frac{g_{m1}}{s C_L} = -\frac{\omega_u}{s} \quad \omega_u = A_v(0) \cdot p_1 = \frac{g_{m1}}{C_L}
\]
Cascode amplifiers

The second pole is approximately at

\[ p_2 = -\frac{g_{o1} + Y_{in2}}{C_x} \]

\( Y_{in2} \) is the resistance looking into the M2's source at high frequencies.

\[ Y_{in2} = g'_{m2} - g_{o2} \left( \frac{V_o}{V_{s2}} - 1 \right) \]

- At frequencies \( \omega \gg \frac{(g_{o2} + G_L)}{C_L} \),

\[ \frac{V_o}{V_{s2}} = \frac{g'_{m2} + g_{o2}}{g_{o2} + G_L + sC_L} \approx \frac{g'_{m2}}{sC_L} \Rightarrow Y_{in2} \approx g'_{m2} \left( 1 - \frac{g_{o2}}{sC_L} \right) + g_{o2} \approx g'_{m2} \]

\[ p_2 \approx -\frac{g'_{m2}}{C_x} \approx -\frac{g_{m2}}{K C_{gs2}} \approx -\frac{\omega_T}{K} \]

where \( K \) is between 1 and 2 (usually closer to 1).
Differential amplifier

Assume
- $M1 = M2$.
- $R_{D1} = R_{D2} = R_D$.
- Neglect $r_o$.
- $R_{SS} \to \infty$.

\[ V_{id} = V_{i1} - V_{i2} \quad I_{dd} = I_{d1} - I_{d2} \quad V_{od} = V_{o1} - V_{o2} = -I_{dd}R_D \]

Assume $M1$ and $M2$ are in the saturation region,

\[ I_{d1} = \frac{1}{2}k(V_{gs1} - V_t)^2 \quad I_{d2} = \frac{1}{2}k(V_{gs2} - V_t)^2 \quad k = \mu C_{ox} \frac{W}{L} \]
Differential amplifier – large-signal analysis

Summing currents at the common source node, we have

\[ I_{d1} + I_{d2} = I_{SS} \quad \Rightarrow \quad I_{d1} = \frac{I_{SS}}{2} + \frac{I_{dd}}{2} \quad I_{d2} = \frac{I_{SS}}{2} - \frac{I_{dd}}{2} \]

The gate voltages can be written as

\[ V_{gs1} = V_t + \sqrt{\frac{2I_{d1}}{K}} \quad V_{gs2} = V_t + \sqrt{\frac{2I_{d2}}{K}} \]

The differential input voltage is

\[ V_{id} = V_{gs1} - V_{gs2} = \sqrt{\frac{2I_{d1}}{k}} - \sqrt{\frac{2I_{d2}}{k}} = \frac{2}{k} \left( \sqrt{I_{d1}} - \sqrt{I_{d2}} \right) \]

Squaring

\[ V_{id}^2 = \frac{2}{k} \left[ I_{d1} + I_{d2} - 2\sqrt{I_{d1}I_{d2}} \right] = \frac{2}{k} \left[ I_{SS} - \sqrt{I_{SS}^2 - I_{dd}^2} \right] \]
Differential amplifier – large-signal analysis

Rearrange, then we have

\[ I_{dd} = \frac{k}{2} V_{id} \sqrt{\frac{4I_{SS}}{k}} - V_{id}^2 \quad \text{and} \quad I_{d1} = \frac{I_{SS}}{2} + \frac{I_{dd}}{2} \quad I_{d2} = \frac{I_{SS}}{2} - \frac{I_{dd}}{2} \]

Define \( V_{IM} \) as the differential input voltage at which one of the MOST is turned off, i.e.,

\[ I_{SS} = \frac{k}{2} V_{IM} \sqrt{\frac{4I_{SS}}{k}} - V_{IM}^2 \quad \Rightarrow \quad V_{IM} = \sqrt{\frac{2I_{SS}}{k}} = \sqrt{2} \left( V_{ov1} \right)_{V_{id}=0} = \sqrt{2} \left( V_{ov2} \right)_{V_{id}=0} \]
Differential- and common-mode small-signal analysis

The small-signal performance of a differential amplifier can be separated into a differential mode and common mode analysis. This separation allows us to take advantage of the following simplifications.

Half-Circuit Concept:

Note: The half-circuit concept is valid as long as the resistance seen looking into each source is approximately the same.
Differential-mode analysis

Small Signal Model:

Miller Approach:
Assume that \( R_L < (1/\omega C_f) \), then \( v_{out} \approx -g_m R_L v_1 \)
Therefore, the small-signal model can be approximated as,

where
\[
C_m = C_f(1+g_m R_L)
\]
Differential-mode analysis

The small-signal analysis of the previous circuit defining \( C_t = C_t + C_m \) is,

\[
\frac{v_{out}}{v_{in}} = \left( \frac{v_{out}}{v_1} \right) \left( \frac{v_1}{v_{in}} \right) = (-g_m R_L) \left( \frac{r_\pi}{1 + r_\pi C_t s} \right) = -g_m R_L \left( \frac{r_\pi}{r_\pi + R_I + r_b} \right) \left( \frac{1}{s r_\pi C_t (R_I + r_b)} \right)
\]

Therefore we see that the gain (\( K \)), pole (\( p_1 \)), and -3dB frequency (\( \omega_{-3dB} \)) is given as,

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<th>( K )</th>
<th>( p_1 )</th>
<th>( \omega_{-3dB} )</th>
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<tr>
<td>BJT</td>
<td>(-g_m R_L \left( \frac{r_\pi}{r_\pi + R_I + r_b} \right))</td>
<td>( \frac{r_\pi + R_I + r_b}{-r_\pi C_t (R_I + r_b)} )</td>
<td>( \frac{r_\pi + R_I + r_b}{r_\pi C_t (R_I + r_b)} )</td>
</tr>
<tr>
<td>MOS</td>
<td>(-g_m R_L )</td>
<td>( \frac{1}{C_t R_I} )</td>
<td>( \frac{1}{C_t R_I} )</td>
</tr>
</tbody>
</table>
Common-mode analysis

Assumptions: Tail capacitance is dominant and self-resistance is negligible.

\[ A_{cm} = \frac{v_{oc}}{v_{ic}} \approx -\frac{R_L}{Z_T} \quad \text{where} \quad Z_T = \frac{2R_T}{1+sR_TC_T} \quad \Rightarrow \quad A_{cm} = \frac{v_{oc}}{v_{ic}} \approx -\frac{R_L}{2R_T(1+sR_TC_T)} \]

This zero at \( \omega = 1/R_TC_T \) causes the CM gain to increase, resulting in a \( CMRR \) decrease.

\[ CMRR = \left| \frac{A_{dm}}{A_{cm}} \right| = \frac{\left( \frac{g_m2R_Tr_\pi}{r_\pi+R_I+r_b} \right)\left( \frac{1}{sr_\pi C_t(R_I+r_b)} \right)}{(1+sR_TC_T)} = \frac{\left( \frac{g_m2R_Tr_\pi}{r_\pi+R_I+r_b} \right)}{(1+s/\omega_T)(1+s/p_1)} \]
CMRR frequency response
Current-mirrors

\[ I_{IN} = \frac{1}{2} k' \left( \frac{W}{L} \right)_1 (V_R - V_{t1})^2 (1 + \lambda_1 V_{D1}) \]

\[ I_{D2} = \frac{1}{2} k' \left( \frac{W}{L} \right)_2 (V_R - V_{t2})^2 (1 + \lambda_2 V_{D2}) \]

\[ I_{D3} = \frac{1}{2} k' \left( \frac{W}{L} \right)_3 (V_R - V_{t3})^2 (1 + \lambda_3 V_{D3}) \]

Let \( V_{t1} = V_{t2} = V_t \) and \( \lambda_1 = \lambda_2 = \lambda \), then

\[ I_{D2} = I_{IN} \cdot \left( \frac{W/L}{W/L} \right)_2 \cdot \frac{1 + \lambda V_{D2}}{1 + \lambda V_{D1}} = I_{IN} \cdot \left( \frac{W/L}{W/L} \right)_1 \cdot (1 + \varepsilon) \quad \varepsilon \approx \lambda (V_{D2} - V_{D1}) = \frac{V_{D2} - V_{D1}}{V_A} \]

\[ R_{o2} = r_{o2} = \frac{1}{\lambda_2 I_{D2}} \quad V_{o2(min)} = V_{ov2} \approx V_{ov1} \approx \sqrt{\frac{2I_{IN}}{k'(W/L)_1}} \quad V_{DD(min)} = V_{GS1} = V_t + V_{ov1} \]
Cascode current mirror

\[ R_c = r_{o1}r_{o2}(g_m + g_{mb} + g_{o1} + g_{o2}) \approx r_{o1}r_{o2}g_m(1 + \chi) \]

\[ V_{o(min)} = V_{DS1} + V_{DSAT2} = V_{t3} + V_{ov3} + V_{ov2} \]

\[ V_{DD(min)} = V_{GS3} + V_{GS4} = V_{t3} + V_{t4} + V_{ov3} + V_{ov4} \]

\[ \varepsilon \approx 0 \]
High-swing cascode current mirror

\[ V_1 = V_t + V_{ov} \]
\[ V_2 = V_t + 2V_{ov} \]
\[ V_3 = V_{ov} \]
\[ V_4 = V_{ov} \]

\[ V_{o(min)} = V_{DS1} + V_{DSAT2} = 2V_{ov} \]
\[ V_{DD(min)} = V_2 = V_t + 2V_{ov} \]
\[ \epsilon = 0 \]

- In practice, select \((W/L)_5 < (1/4)(W/L)\) due to body effect and design margin.
- To bias M2 and M4 in the active region, want \(V_2 - V_1 < V_t\) \(\Rightarrow\) \(V_{ov} < V_t\).
Common-source with current source load

\[ G'_L = g_{o1} + g_{o2} \quad C'_L = C_L + C_{o2} \]

- The \( V_o \) range in normal operation is between \( V_{DSAT1} \) and \( V_{DD} - V_{o2(min)} \).
Common-source amplifier with active loads