Review of MOSFET modeling

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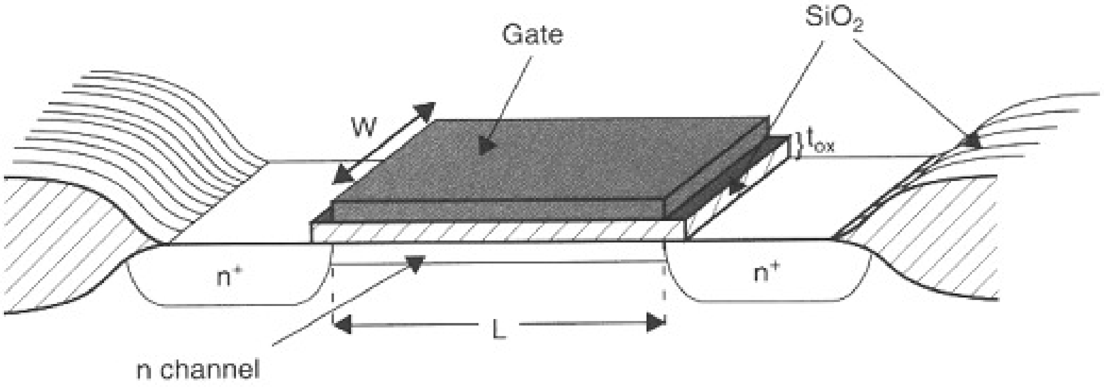
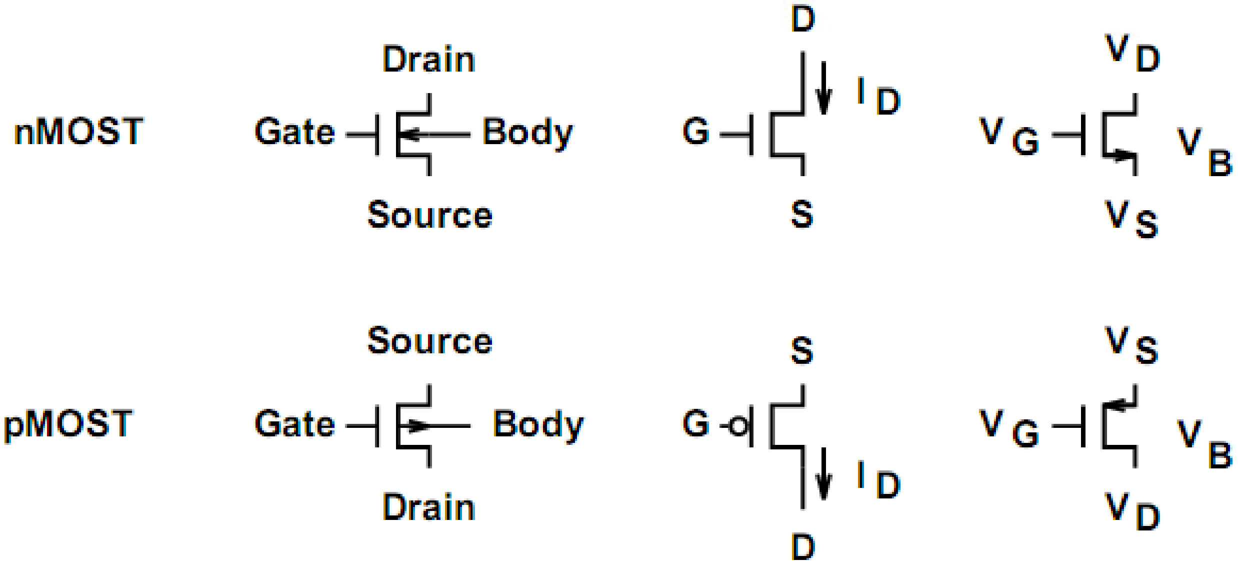
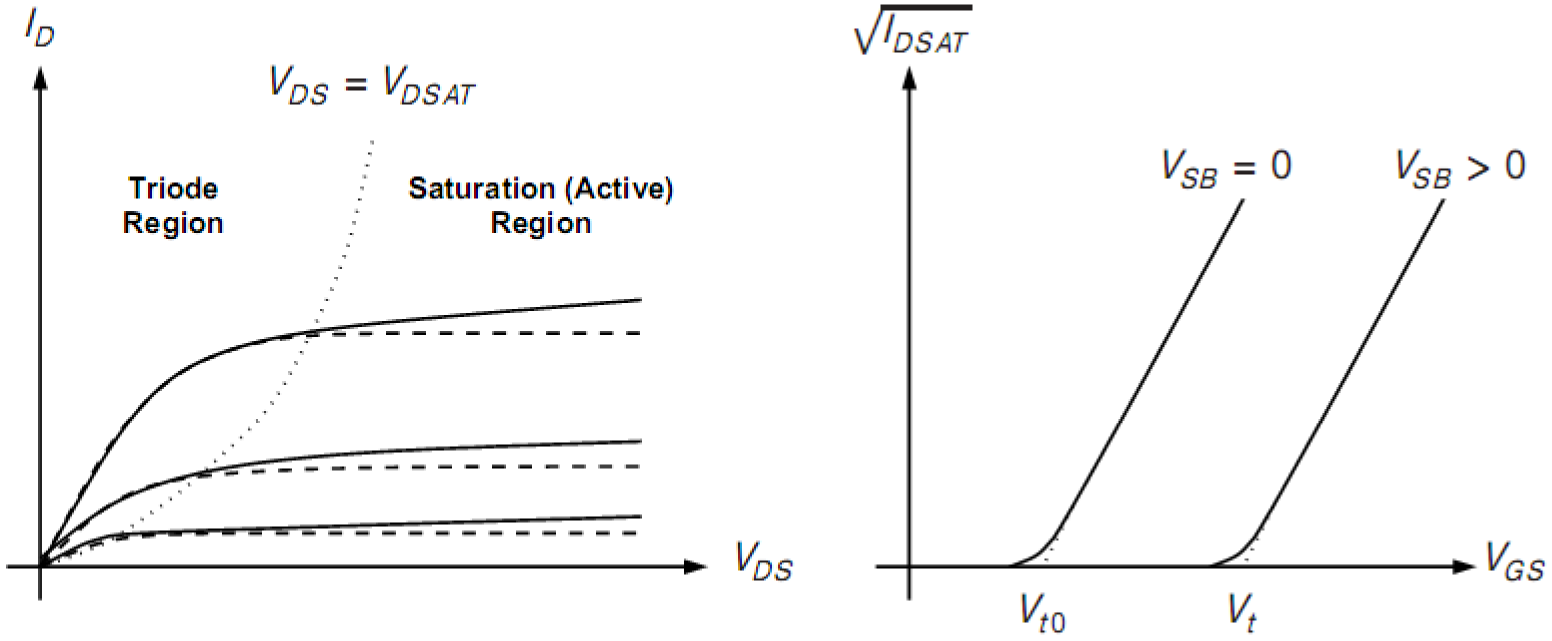
MOS Transistor





MOS I-V characteristics



$$I_D = \mu C_{ox}\frac{W}{L}\left[(V_{GS}-V_t)V_{DS}-\frac{1}{2}V_{DS}^2\right] \qquad \text{for} \quad V_{DS} \le V_{DSAT} = V_{GS}-V_t$$

$$I_{DSAT} = I_D \;@\; V_{DS} = V_{DSAT} = \frac{1}{2}\mu C_{ox}\frac{W}{L}(V_{GS}-V_t)^2$$

MOSFET's Threshold Voltage

$$V_t = V_{t0} + \gamma\left[\sqrt{V_{SB} + 2\phi_f} - \sqrt{2\phi_f}\right] \quad \text{for} \quad V_{SB} > 0$$

V_{t0} is the threshold voltage when $V_{SB} = 0$.

$$V_{t0} = 2\phi_f + \gamma\sqrt{2\phi_f} + V_{FB} \quad \phi_f = \frac{kT}{q}\ln\left(\frac{N_{SUB}}{n_i}\right) \quad \gamma = \frac{\sqrt{2q\epsilon_{si}N_{SUB}}}{C_{ox}} \quad C_{ox} = \frac{\epsilon_{ox}}{t_{ox}}$$

The Fermi level ϕ_f is temperature dependent, i.e.,

$$\frac{d\phi_f}{dT} = -\frac{1}{T}\left[\frac{E_{g0}}{2q} - \phi_f\right] \qquad E_{g0} = \text{Silicon band gap at } T = 0^{\circ}\text{K}$$

The V_{t0}'s temperature coefficient is

$$\frac{dV_{t0}}{dT} = -\frac{1}{T}\left[\frac{E_{g0}}{2q} - \phi_f\right]\left[2 + \frac{\gamma}{\sqrt{2\phi_f}}\right]$$

- dV_{t0}/dT is usually in the range between −0.5 mV/ °C to −4 mV/ °C.

Square-law I-V characteristic

In triode region, 1st-order long-channel model is

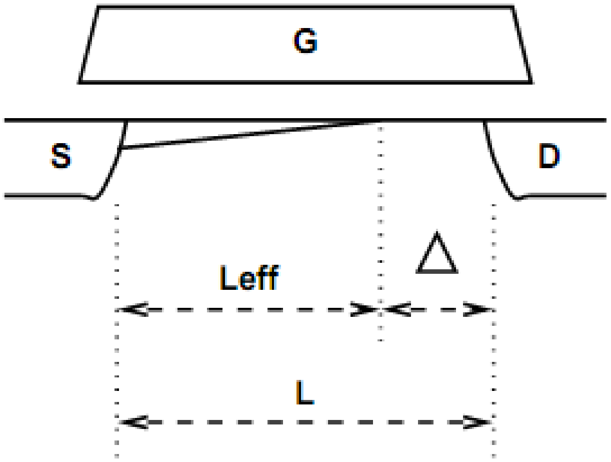
$$I_D = \mu C_{ox}\frac{W}{L}\left[(V_{GS}-V_t)V_{DS}-\frac{1}{2}V_{DS}^2\right] = k'\frac{W}{L}\left[(V_{GS}-V_t)V_{DS}-\frac{1}{2}V_{DS}^2\right]$$

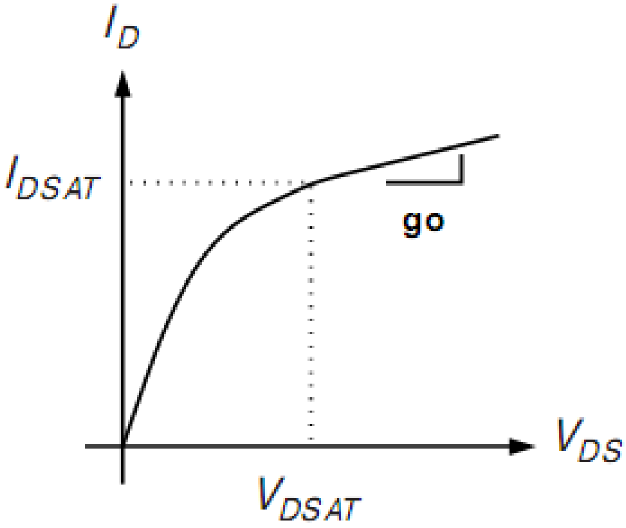
When $V_{DS} \geq V_{DSAT} = V_{GS} - V_t$, the MOST is in the *pinch-off* region (or *saturation* region),

$$I_{DS} = I_{DSAT} = I_D(V_{DS} = V_{GS} - V_t) = \frac{1}{2}\mu C_{ox}\frac{W}{L}(V_{GS}-V_t)^2 = \frac{1}{2}k'\frac{W}{L}V_{ov}^2$$

- $k' = \mu C_{ox}$ is called the *process transconductance parameter.*
- $k = \beta = \mu C_{ox}\frac{W}{L}$ is called the *device transconductance parameter.*
- $V_{ov} = V_{GS} - V_t$ is called the *gate drive* or the *overdrive*.

Channel-length modulation





$$I_{D(sat)} = \frac{1}{2}k'\frac{W}{L_{eff}}V_{ov}^2 \qquad L_{eff} = L - \Delta \qquad \Delta V_{DS} = V_{DS} - V_{DSAT}$$

Using one-dimensional abrupt PN junction model,

$$\Delta \approx \sqrt{\frac{2\epsilon_{si}}{qN_{SUB}}}\sqrt{V_{DS} - V_{DSAT} + \Psi_o}$$

Channel-length modulation

The I_D variation due to V_{DS} can be written as:

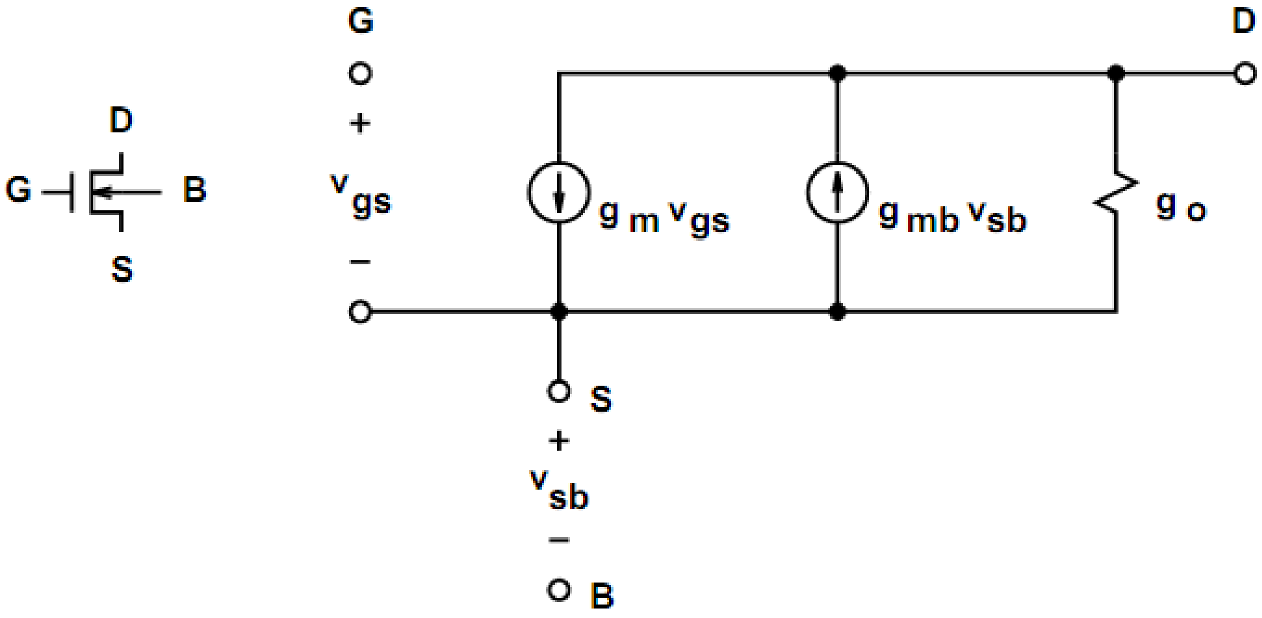
$$\frac{\partial I_D}{\partial V_{DS}} = \frac{\partial I_D}{\partial L_{eff}} \times \frac{\partial L_{eff}}{\partial V_{DS}} = -\frac{I_D}{L_{eff}} \times -\frac{1}{2}\sqrt{\frac{2\epsilon_{si}}{qN_{SUB}}}\frac{1}{\sqrt{V_{DS} - V_{DSAT} + \Psi_o}} = I_D \cdot \lambda$$

The drain current in the pinch-off region can be approximated as

$$I_{D(sat)} = \frac{1}{2}k'\frac{W}{L}V_{ov}^2(1 + \lambda V_{DS}) = \frac{1}{2}k'\frac{W}{L}V_{ov}^2\left(1 + \frac{V_{DS}}{V_A}\right)$$

- λ is inversely proportional to L, i.e., $\lambda \propto 1/L$.
- Typical values of λ are in the range 0.05 V^{-1} to 0.005 V^{-1}.
- The accurate calculation of λ from the device structure is quite difficult. Extraction from experimental data is usually necessary.

Small-signal model of MOSFET in saturation region



$$\text{Transconductance} = g_m \equiv \frac{\partial I_D}{\partial V_{GS}} = k'\frac{W}{L}V_{ov}(1+\lambda V_{DS}) = \sqrt{2k'\frac{W}{L}I_D(1+\lambda V_{DS})} = \frac{I_D}{V_{ov}/2}$$

$$\text{Output Conductance} = g_o \equiv \frac{\partial I_D}{\partial V_{DS}} = \lambda I_D$$

Small-signal model of MOSFET in saturation region

$$\text{Body Transconductance} = g_{mb} \equiv -\frac{\partial I_D}{\partial V_{SB}} = -\frac{\partial I_D}{\partial V_t} \times \frac{\partial V_t}{\partial V_{SB}} = g_m \times \frac{\gamma}{2\sqrt{V_{SB} + 2\phi_f}}$$

Thus

$$g_{mb} = g_m \times \chi \qquad \text{where} \quad \chi \equiv \frac{\gamma}{2\sqrt{V_{SB} + 2\phi_f}}$$

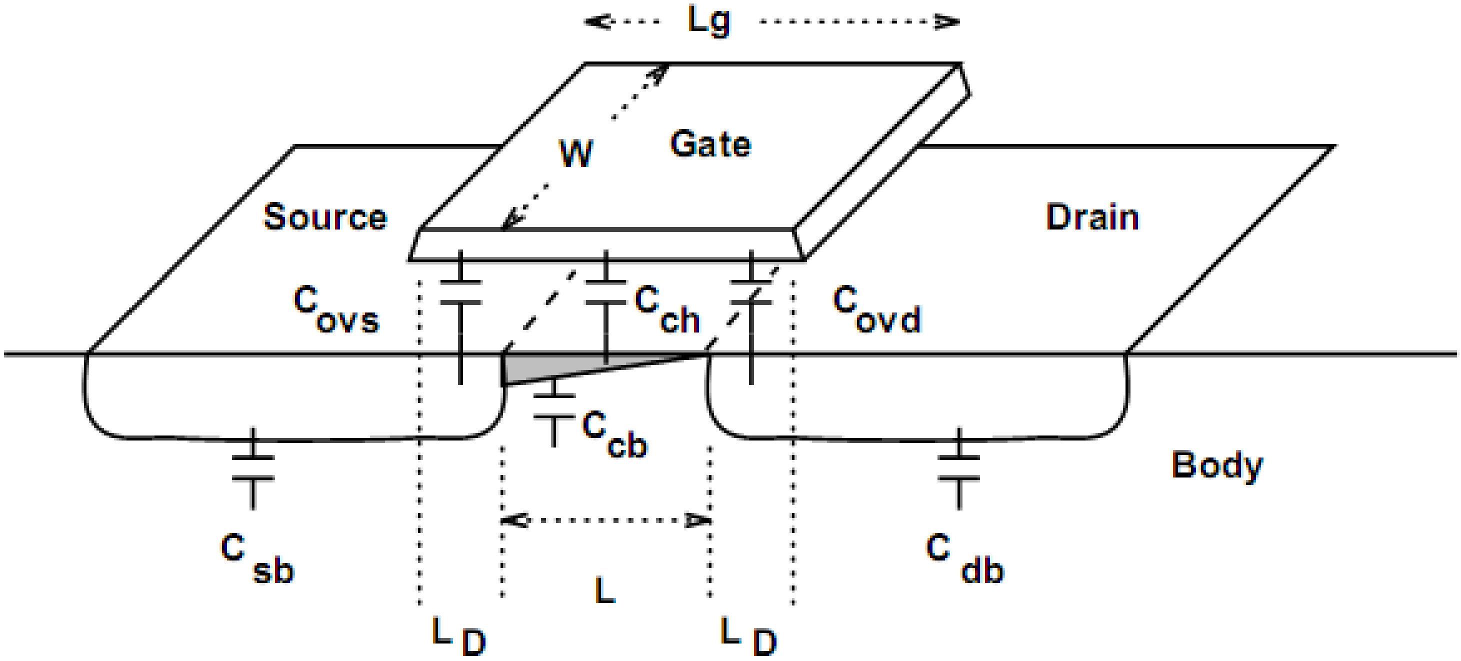
- The factor χ is typically 0.1–0.3.

- Since $\gamma = \sqrt{2q\epsilon_{si}N_{SUB}}/C_{ox}$

$$\chi = \left[\epsilon_{si}/\sqrt{\frac{2\epsilon_{si}(V_{SB} + 2\phi_f)}{qN_{SUB}}}\right]\frac{1}{C_{ox}} = \frac{\epsilon_{si}/x_{d\,max}}{C_{ox}} = \frac{C_{depl}}{C_{ox}}$$

$x_{d\,max}$: The width of depletion layer under channel.
C_{depl}: The capacitance/area of depletion layer under channel.

MOSFET small-signal capacitances



$$C_{sb} = AS \times C_J(V_{SB}) + PS \times C_{JSW}(V_{SB}) \qquad C_{db} = AD \times C_J(V_{DB}) + PD \times C_{JSW}(V_{DB})$$

$$C'_{sb} = C_{sb} + C_{cb} \qquad C_{cb} \approx WL \times C_J(V_{SB})$$

- AS and AD are the areas of the source/drain junctions.
- PS and PD are the source/drain perimeters excluding the sides adjacent to channel.

MOSFET small-signal capacitances

Junction Capacitances:

$$C_{sb} = \frac{C_{sbo}}{\left(1 + \frac{V_{SB}}{\psi_o}\right)^m} \qquad C_{db} = \frac{C_{dbo}}{\left(1 + \frac{V_{DB}}{\psi_o}\right)^m} \qquad m = \frac{1}{3} \sim \frac{1}{2}$$

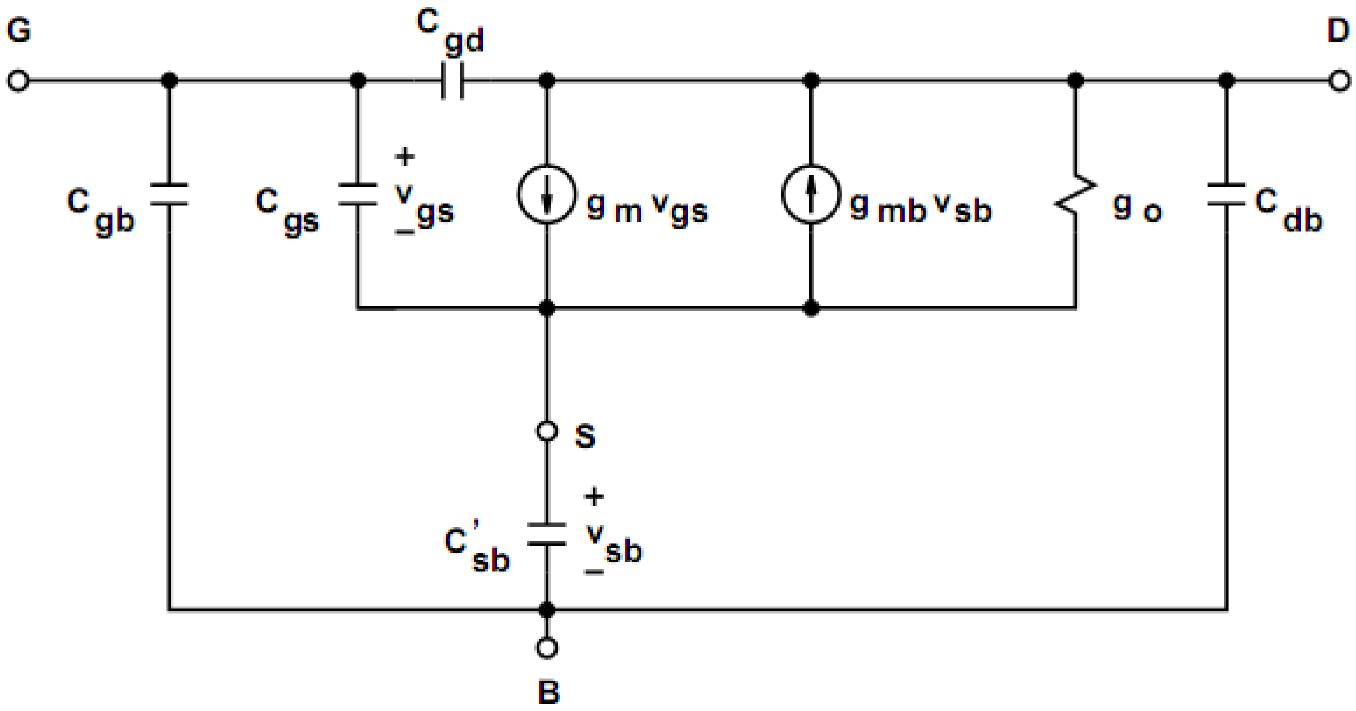
Overlap Capacitances:

$$C_{ovs} = W \times C_{GSO} = W \times (nL_D C_{ox}) \qquad C_{ovd} = W \times C_{GDO} = W \times (nL_D C_{ox})$$

$1 \leq n \leq 2$ (Due to friniging)

Channel Capacitance $= C_{ch} \equiv \frac{\partial Q_T}{\partial V_{GS}} = \frac{2}{3} WLC_{ox}$

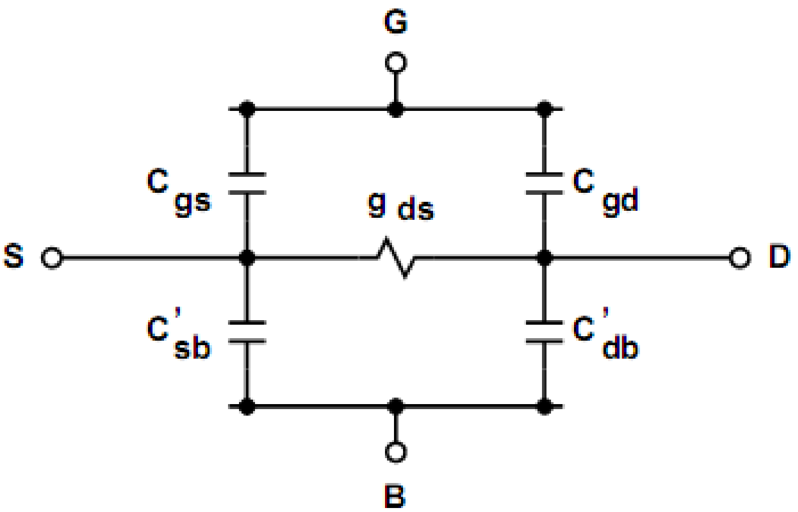
Complete small-signal of MOSFET in saturation region

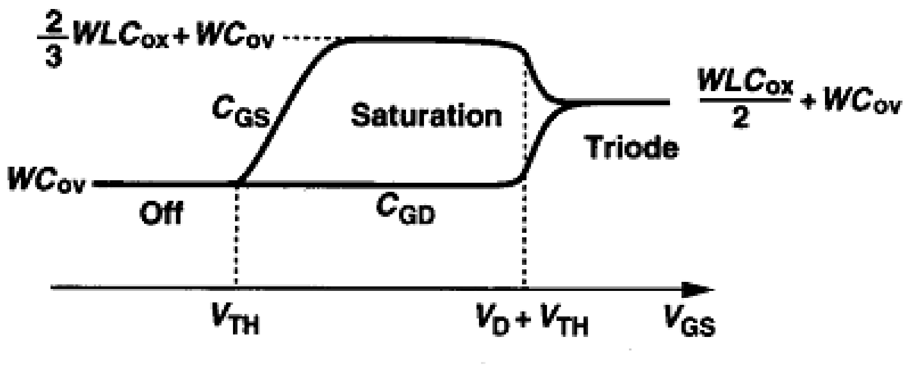


$$C_{gd} = C_{ovd} \qquad C_{gs} = C_{ovs} + \frac{2}{3}WLC_{ox}$$

$$C'_{sb} = C_{sb} + C_{cb} = (AS + W \cdot L) \times C_J(V_{SB}) + PS \times C_{JSW}(V_{SB})$$

Small-signal model of MOSFET in Triode region



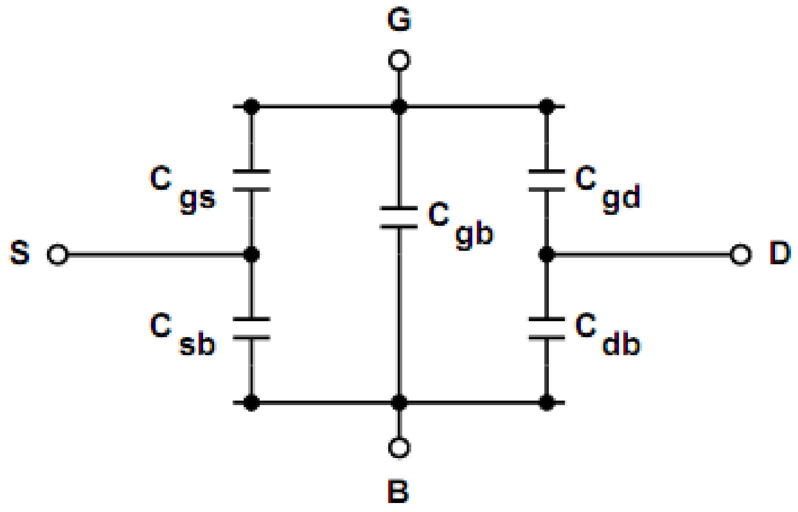


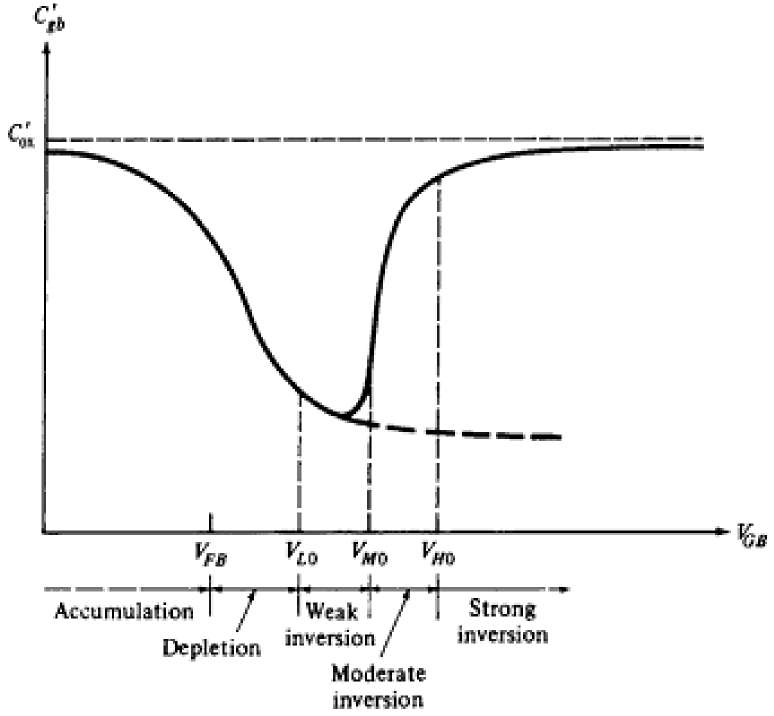
$$g_{ds} = \frac{\partial I_D}{\partial V_{DS}} = \mu C_{ox}\frac{W}{L}(V_{GS} - V_t) \text{ for } V_{DS} \rightarrow 0$$

$$C_{gs} = C_{ovs} + \frac{1}{2}WLC_{ox} \quad C_{gd} = C_{ovd} + \frac{1}{2}WLC_{ox}$$

$$C'_{sb} = C_{sb} + \frac{1}{2}WLC_J(V_{SB}) \quad C'_{db} = C_{db} + \frac{1}{2}WLC_J(V_{DB})$$

Small-signal model of MOSFET in Cut-off region

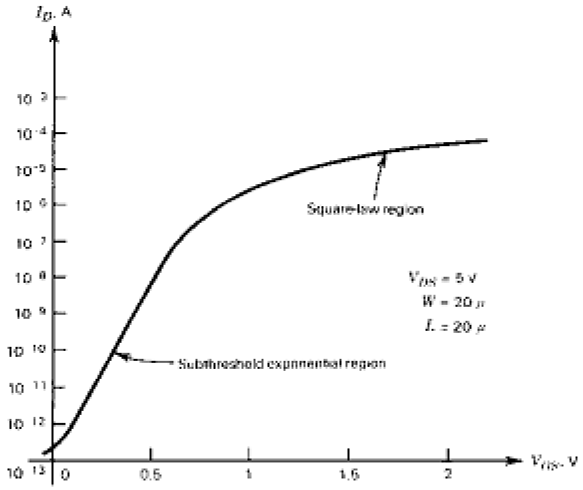


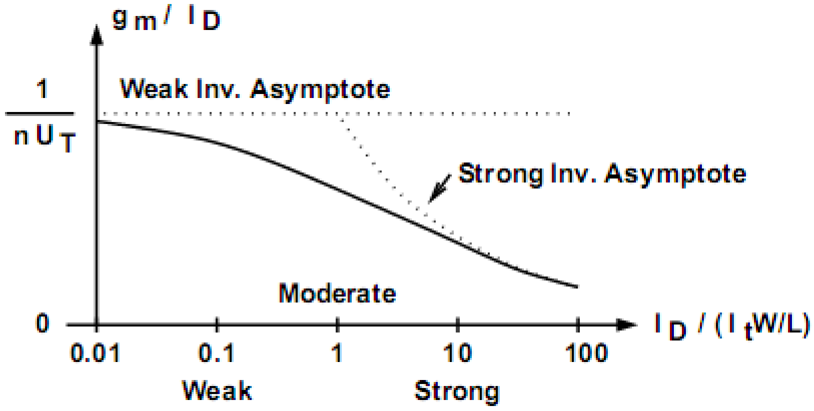


$$C_{gs} = C_{ovs} \qquad C_{gd} = C_{ovd}$$

- C_{gb} is highly nonlinear and dependent on the gate voltage.

MOSFET in subthreshold region





In the weak inversion region

$$I_D = I_t \frac{W}{L} e^{V_{ov}/(nU_T)} \left(1 - e^{-V_{DS}/U_T}\right) \qquad n = \frac{C_{ox} + C_{depl}}{C_{ox}} = 1 + \chi \approx 1.5$$

- $I_t \propto D_n n_{po}$ depends on process parameters (e.g., 20 nA).

MOSFET in subthreshold region

When $|V_{DS}| > 3U_T$, I_D saturates and

$$g_m \equiv \frac{\partial I_D}{\partial V_{GS}} = \frac{I_D}{nU_T} = \frac{I_D}{U_T}\frac{C_{ox}}{C_{ox} + C_{depl}} \qquad \frac{g_m}{I_D} = \frac{1}{nU_T} = \frac{1}{U_T}\frac{C_{ox}}{C_{ox} + C_{depl}}$$

To find V_{ov} for strong inversion, let

$$\frac{g_m}{I_D} = \frac{1}{nU_T} = \frac{2}{V_{ov}} \qquad \Rightarrow \qquad V_{ov} = 2nU_T \approx 78 \text{ mV}$$

- $2nU_T < V_{ov}$ → *Strong Inversion*
  $0 < V_{ov} < 2nU_T$ → *Moderate Inversion*
  $V_{ov} < 0$ → *Weak Inversion*
- In weak inversion, $C_{gs} \simeq C_{gd} \simeq 0$, and

$$C_{gb} = WL \times (C_{ox} \| C_{depl}) = WL \times C_{ox}C_{depl}/(C_{ox} + C_{depl})$$