Current and Voltage References

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Sensitivity and temperature coefficient

- The *sensitivity* of a parameter y to a second one x is defined as

$$S_x^y \equiv \frac{\left(\frac{\Delta y}{y}\right)}{\left(\frac{\Delta x}{x}\right)} = \frac{x}{y} \cdot \frac{\partial y}{\partial x}$$

- The variation of a parameter y that results from changes in temperature is usually characterized by its *fractional temperature coefficient*, which is defined as the fractional change per degree centigrade change in temperature.

$$\mathrm{TC}_y \equiv \frac{\left(\frac{\Delta y}{y}\right)}{\Delta T} = \frac{1}{y} \cdot \frac{\partial y}{\partial T}$$

$$TC_F = \frac{1}{V_{REF}}\left(\frac{\partial V_{REF}}{\partial T}\right) = \frac{1}{T} S_T^{V_{REF}} \text{ parts per million per °C or ppm/°C}$$

Temperature dependence

Temperature dependence of PN junctions:

$$i \approx I_s \exp\left(\frac{v}{V_t}\right)$$

$$I_s = KT^3 \exp\left(\frac{-V_{GO}}{V_t}\right)$$

$$\frac{1}{I_s}\left(\frac{\partial I_s}{\partial T}\right) = \frac{\partial(\ln I_s)}{\partial T} = \frac{3}{T} + \frac{V_{GO}}{TV_t} \approx \frac{V_{GO}}{TV_t}$$

$$\frac{dv_{BE}}{dT} \approx \frac{V_{BE} - V_{GO}}{T} = -2\text{mV/°C at room temperature}$$

(V_{GO} = 1.205 V at room temperature and is called the bandgap voltage)

Temperature dependence of MOSFET in strong inversion:

$$\frac{dv_{GS}}{dT} = \frac{dV_T}{dT} + \sqrt{\frac{2L}{WC_{ox}}}\frac{d}{dT}\left(\sqrt{\frac{i_D}{\mu_o}}\right)$$

$$\mu_o = KT^{-1.5}$$

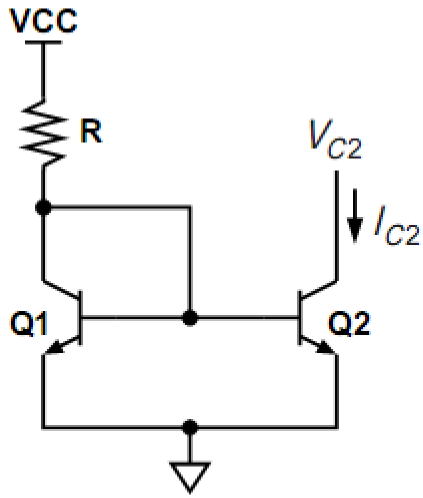
$$V_T(T) = V_T(T_o) - \alpha(T-T_o)$$

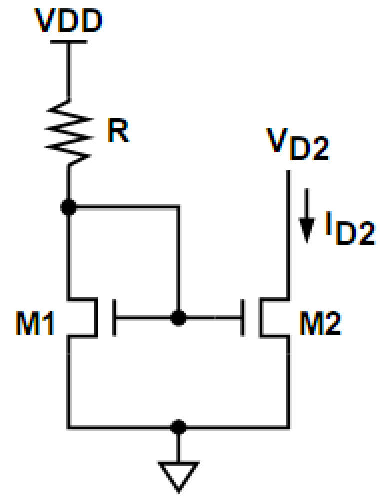
$$\frac{dv_{GS}}{dT} \approx -\alpha \approx -2.3\frac{\text{mV}}{°\text{C}}$$

Resistors:

$(1/R)(dR/dT)$ ppm/°C

Simple current source



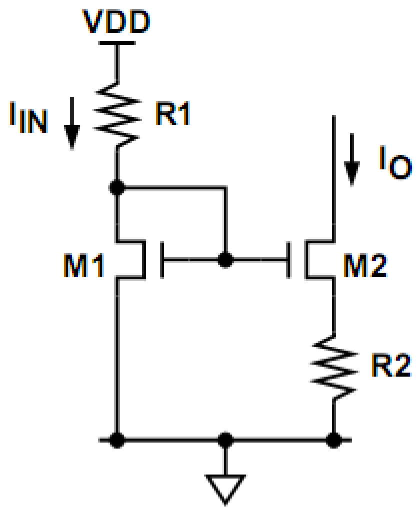


$$I_{C2} \approx I_{C1} \approx \frac{V_{CC} - V_{BE1(on)}}{R} \approx \frac{V_{CC}}{R}$$

$$S_{V_{CC}}^{I_{C2}} = \frac{V_{CC}}{I_{C2}} \cdot \frac{\partial I_{C2}}{\partial V_{CC}} = \frac{V_{CC}}{V_{CC}/R} \cdot \frac{\partial}{\partial V_{CC}} \left(\frac{V_{CC}}{R}\right) = R \cdot \left(\frac{1}{R}\right) = 1$$

$$\frac{\partial I_{C2}}{\partial T} = \frac{1}{R}\frac{\partial V_{CC}}{\partial T} - \frac{V_{CC}}{R^2}\frac{\partial R}{\partial T} = I_{C2}\left(\frac{1}{V_{CC}}\frac{\partial V_{CC}}{\partial T} - \frac{1}{R}\frac{\partial R}{\partial T}\right) \quad \Rightarrow \quad TC_{I_{C2}} = TC_{V_{CC}} - TC_R$$

Widlar current source



Let $V_A \rightarrow \infty$ and $\gamma \rightarrow 0$,

$$I_{IN} = \frac{V_{DD} - V_{ov1} - V_t}{R_1} = \frac{1}{2}k'\left(\frac{W}{L}\right)_1 V_{ov1}^2 \qquad V_{ov1} = \sqrt{\frac{2I_{IN}}{k'(W/L)_1}}$$

$$V_{ov1} = V_{ov2} + I_O R_2 \quad \Rightarrow \quad I_O R_2 + \sqrt{\frac{2I_O}{k'(W/L)_2}} - V_{ov1} = 0$$

$$\sqrt{I_O} = \frac{1}{2R_2}\left(-\sqrt{\frac{2}{k'(W/L)_2}} + \sqrt{\frac{2}{k'(W/L)_2} + 4R_2 V_{ov1}}\right)$$

$$\frac{1}{2\sqrt{I_O}}\frac{\partial I_O}{\partial V_{DD}} = \frac{1}{4R_2}\frac{1}{\sqrt{\frac{2}{k'(W/L)_2} + 4R_2 V_{ov1}}}4R_2\frac{\partial V_{ov1}}{\partial V_{DD}} \qquad \frac{\partial V_{ov1}}{\partial V_{DD}} = \sqrt{\frac{2}{k'(W/L)_1}}\frac{1}{2\sqrt{I_{IN}}}\frac{\partial I_{IN}}{\partial V_{DD}}$$

$$S_{V_{DD}}^{I_O} = \frac{V_{ov1}}{\sqrt{V_{ov2}^2 + 4I_O R_2 V_{ov1}}} S_{V_{DD}}^{I_{IN}} \approx \frac{V_{ov1}}{\sqrt{4V_{ov1}^2}} S_{V_{DD}}^{I_{IN}} = \frac{1}{2} S_{V_{DD}}^{I_{IN}}$$

Threshold referenced current source

Circuit:

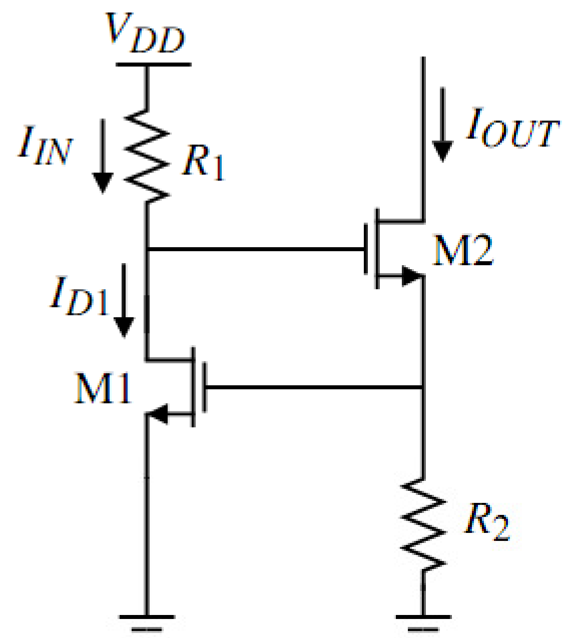


Fig. 360-1

Operation:

$$I_{OUT} = \frac{V_{GS1}}{R_2} = \frac{V_T + \sqrt{\frac{2I_{IN}}{K'(W_1/L_1)}}}{R_2}$$

$$\approx \frac{V_T}{R_2} \text{ if } V_T > V_{ON1}$$

The sensitivity of I_{OUT} with respect to V_{DD} is

$$S_{V_{DD}}^{I_{OUT}} = \left(\frac{V_{ON1}}{I_{OUT}R_2}\right) S_{V_{DD}}^{I_{IN}} = \left(\frac{V_{ON1}}{2V_{GS1}}\right) S_{V_{DD}}^{I_{IN}}$$

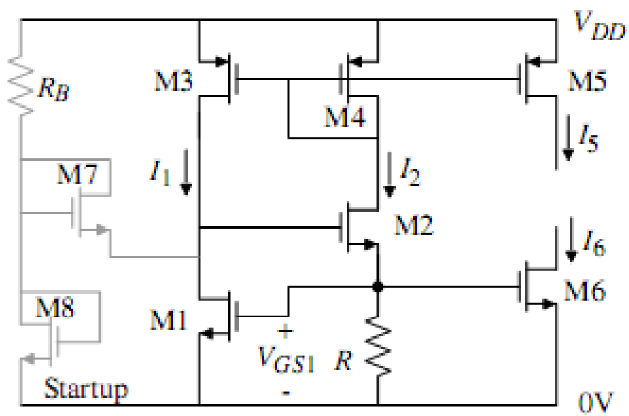
For example, if $V_T = 1$V, $V_{ON1} = 0.1$V and $S_{V_{DD}}^{I_{IN}} \approx 1$, then

$$S_{V_{DD}}^{I_{OUT}} = \left(\frac{0.1}{2 \cdot 1.1}\right) = 0.045$$

Therefore, if V_{DD} changes by 10%, I_{REF} or I_{OUT} changes by 0.45%.

Bootstrapped current source

- Very good power supply independent!



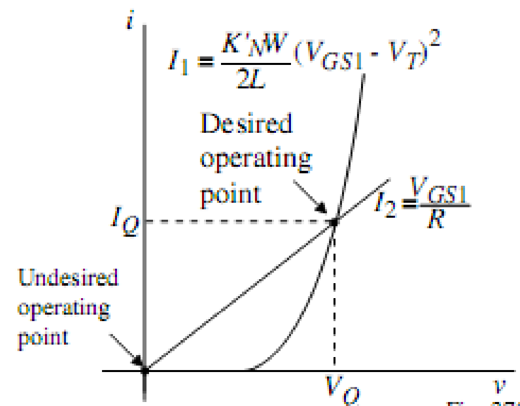


Fig. 370-06

Principle:

If M3 = M4, then $I_1 \approx I_2$. However, the M1-R loop gives $V_{GS1} = V_{T1} + \sqrt{\frac{2I_1}{K_N'(W_1/L_1)}}$

Solving these two equations gives $I_2 = \frac{V_{GS1}}{R} = \frac{V_{T1}}{R} + \left(\frac{1}{R}\right)\sqrt{\frac{2I_1}{K_N'(W_1/L_1)}}$

The output current, $I_{out} = I_1 = I_2$ can be solved as $I_{out} = \frac{V_{T1}}{R} + \frac{1}{\beta_1 R^2} + \frac{1}{R}\sqrt{\frac{2V_{T1}}{\beta_1 R} + \frac{1}{(\beta_1 R)^2}}$

Constant Gm biasing

<u>Technique to Make g_m Dependent on a Resistor</u>

Consider the following circuit with all transistors having a W/L = 10. This is a bootstrapped reference which creates a V_{bias} independent of V_{DD}. The two key equations are:

$$I_3 = I_4 \Rightarrow I_1 = I_2$$

and

$$V_{GS1} = V_{GS2} + I_2R$$

Solving for I_2 gives:

$$I_2 = \frac{V_{GS1}-V_{GS2}}{R} = \frac{1}{R}\left(\sqrt{\frac{2I_1}{\beta_1}} - \sqrt{\frac{2I_2}{\beta_2}}\right) = \frac{\sqrt{2I_1}}{R\sqrt{\beta_1}}\left(1 - \frac{1}{2}\right)$$

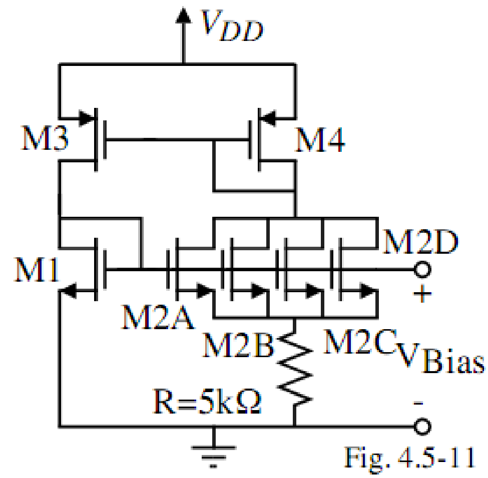


Fig. 4.5-11

$$\therefore \quad \sqrt{I_2} = \frac{1}{R\sqrt{2\beta_1}} \quad \Rightarrow \quad I_2 = I_1 = \frac{1}{2\beta_1 R^2} = \frac{1}{2\cdot 110\text{x}10^{-6}\cdot 10\cdot 25\text{x}10^{6}} = 18.18\mu A$$

Now, V_{bias} can be written as

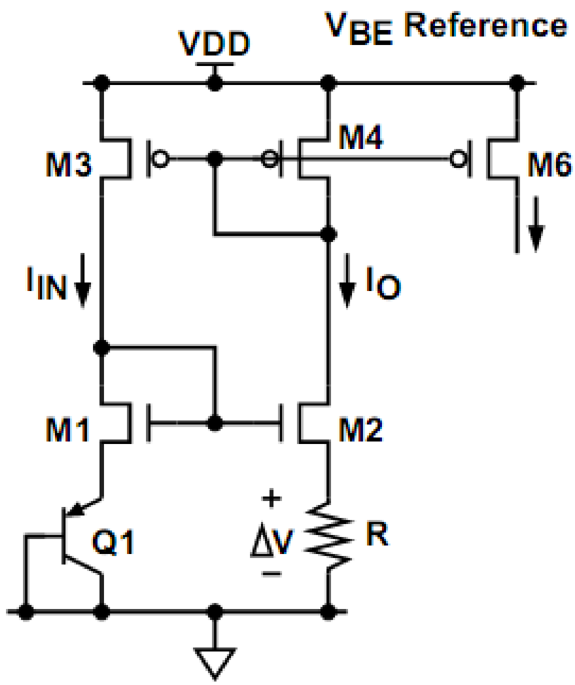
$$V_{bias} = V_{GS1} = \sqrt{\frac{2I_2}{\beta_1}} + V_{TN} = \frac{1}{\beta_1 R} + V_{TN} = \frac{1}{110\text{x}10^{-6}\cdot 10\cdot 5\text{x}10^{3}} + 0.7 = 0.1818 + 0.7 = 0.8818V$$

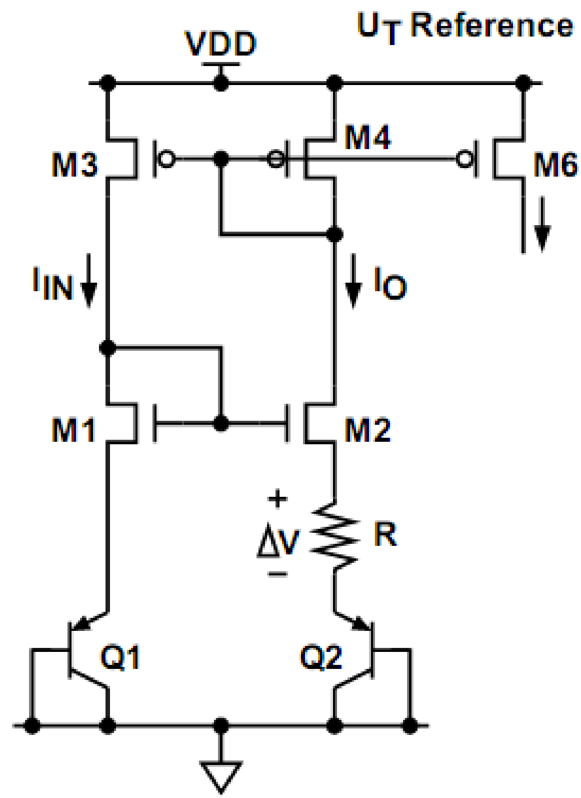
Any transistor with $V_{GS} = V_{bias}$ will have a current flow that is given by $1/2\beta R^2$.

Therefore, $$g_m = \sqrt{2I\beta} = \sqrt{\frac{2\beta}{2\beta R^2}} = \frac{1}{R} \quad \Rightarrow \quad \boxed{g_m = \frac{1}{R}}$$

(This means that the temperature dependence of g_m will be that of 1/R which can be used to achieve temperature controlled performance.)

Bootstrapped current sources



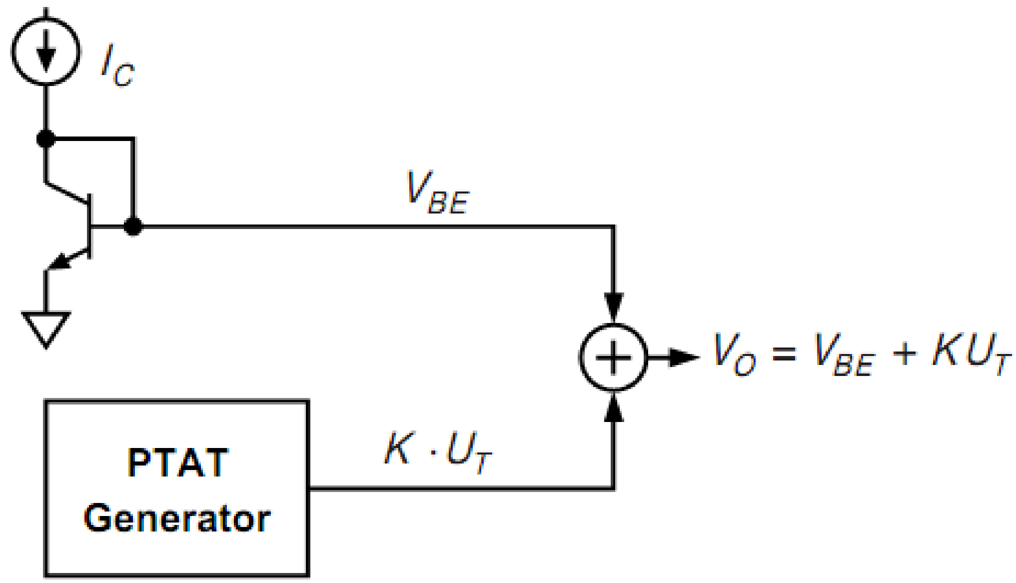


$$\Delta V = V_{BE1}$$

$$\Delta V = U_T \ln\left(\frac{I_{S2}}{I_{S1}} \cdot \frac{(W/L)_3}{(W/L)_4}\right)$$

Also known as PTAT current source

Bandgap reference



- $U_T = kT/q = 26$ mV at $T = 300^\circ$K. $\partial U_T/\partial T = k/q = 0.087$ mV/ $^\circ$C.
- $V_{BE} = 600$ mV at $T = 300^\circ$K. $\partial V_{BE}/\partial T \approx -2$ mV/ $^\circ$C.
- Want $K = 23$ so that $\partial V_o/\partial T = 0$ at 300 $^\circ$K and $V_O \approx 1.2$ V.

Bandgap reference

For a BJT biased in the forward-active region, we have

$$V_{BE} = V_{G0}\left(1-\frac{T}{T_0}\right) + V_{BE0}\frac{T}{T_0} + mU_T\ln\left(\frac{T_0}{T}\right) + U_T\ln\left(\frac{J_C}{J_{C0}}\right) \qquad U_T = \frac{kT}{q}$$

V_{G0}	Bandgap voltage of Si extrapolated to 0°K ($\approx$ 1.206 V)
k	Boltzmann's constant
m	Constant ($\approx$ 2.3)
T_0	Reference temperature
J_C	Collector current density ($= I_C/A_E$)
J_{C0}	Collector current density at T_0

Let

$$V_O = V_{BE} + KU_T \qquad \text{and} \qquad \frac{J_C}{J_{C0}} = \left(\frac{T}{T_0}\right)^{\alpha}$$

We have

$$V_O = V_{G0} + \frac{T}{T_0}(V_{BE0} - V_{G0}) + (m-\alpha)U_T\ln\left(\frac{T_0}{T}\right) + K\cdot U_T$$

Bandgap reference

Then

$$\frac{\partial V_O}{\partial T} = \frac{1}{T_0}(V_{BE0} - V_{G0}) + (m - \alpha)\frac{k}{q}\left[\ln\left(\frac{T_0}{T}\right) - 1\right] + K \cdot \frac{k}{q}$$

Set $\partial V_O/\partial T = 0$ at $T = T_0$, we obtain

$$K = \frac{1}{U_{T0}} \cdot \left[V_{G0} + (m - \alpha)U_{T0} - V_{BE0}\right] \qquad U_{T0} = \frac{kT_0}{q}$$

$$V_O = V_{G0} + U_T(m - \alpha)\left[1 + \ln\left(\frac{T_0}{T}\right)\right] \qquad \frac{\partial V_O}{\partial T} = \frac{k}{q}(m - \alpha)\ln\left(\frac{T_0}{T}\right)$$

- At $T = T_0$,

$$V_O = V_{G0} + U_{T0}(m - \alpha) \qquad \frac{\partial V_O}{\partial T} = 0$$

- If $T_0 = 300^\circ K$ and $\alpha = 1$, then,

$$K = \frac{1.24 - V_{BE0}}{0.0258} \qquad \text{and} \qquad V_O = 1.24\ \text{V} \quad \text{at } T = T_0$$

Bandgap reference voltage vs. Temp.

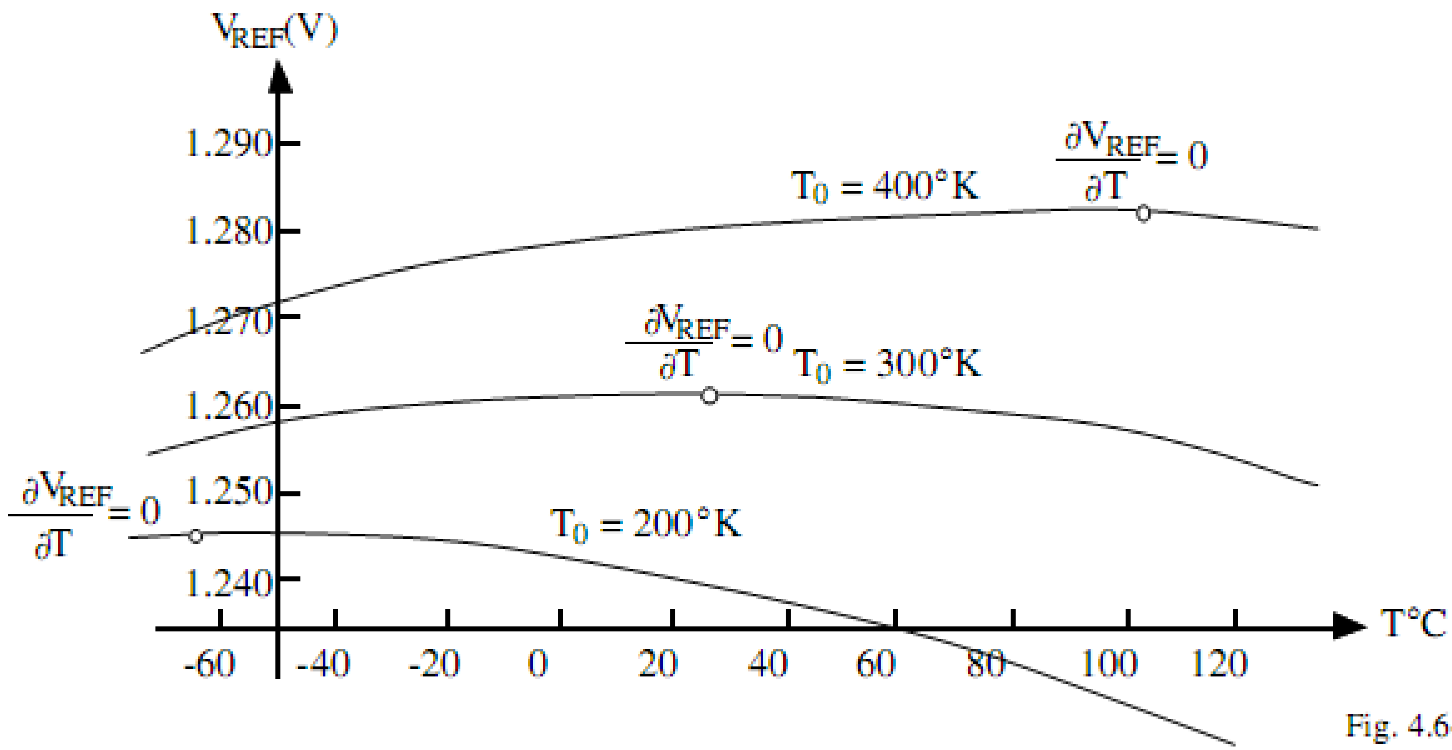
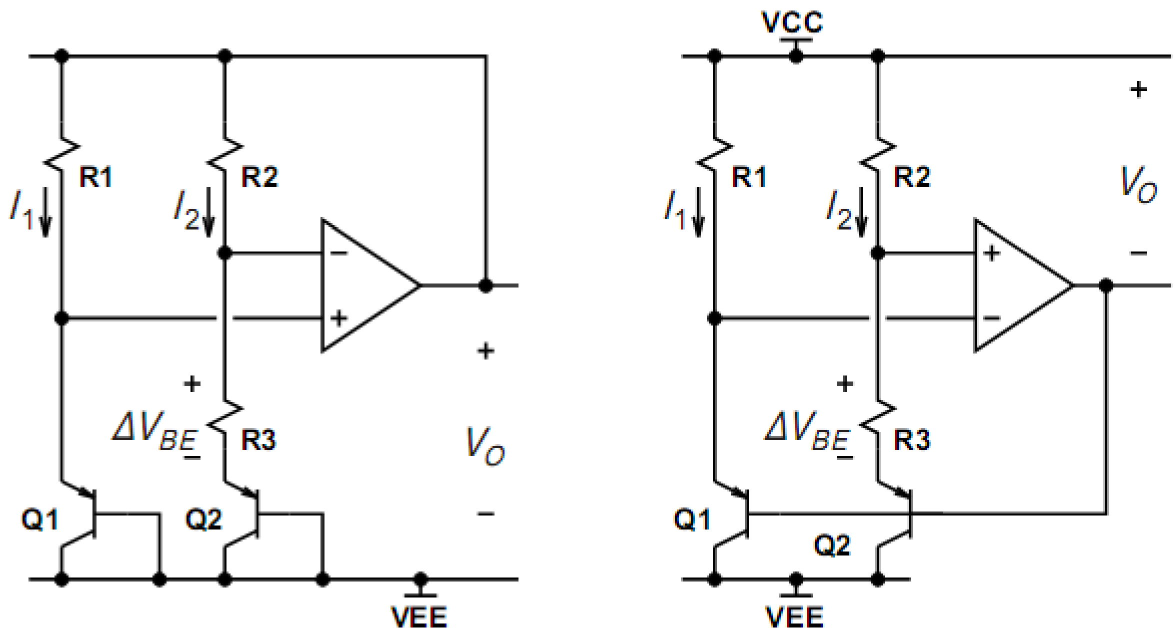


Fig. 4.6-3

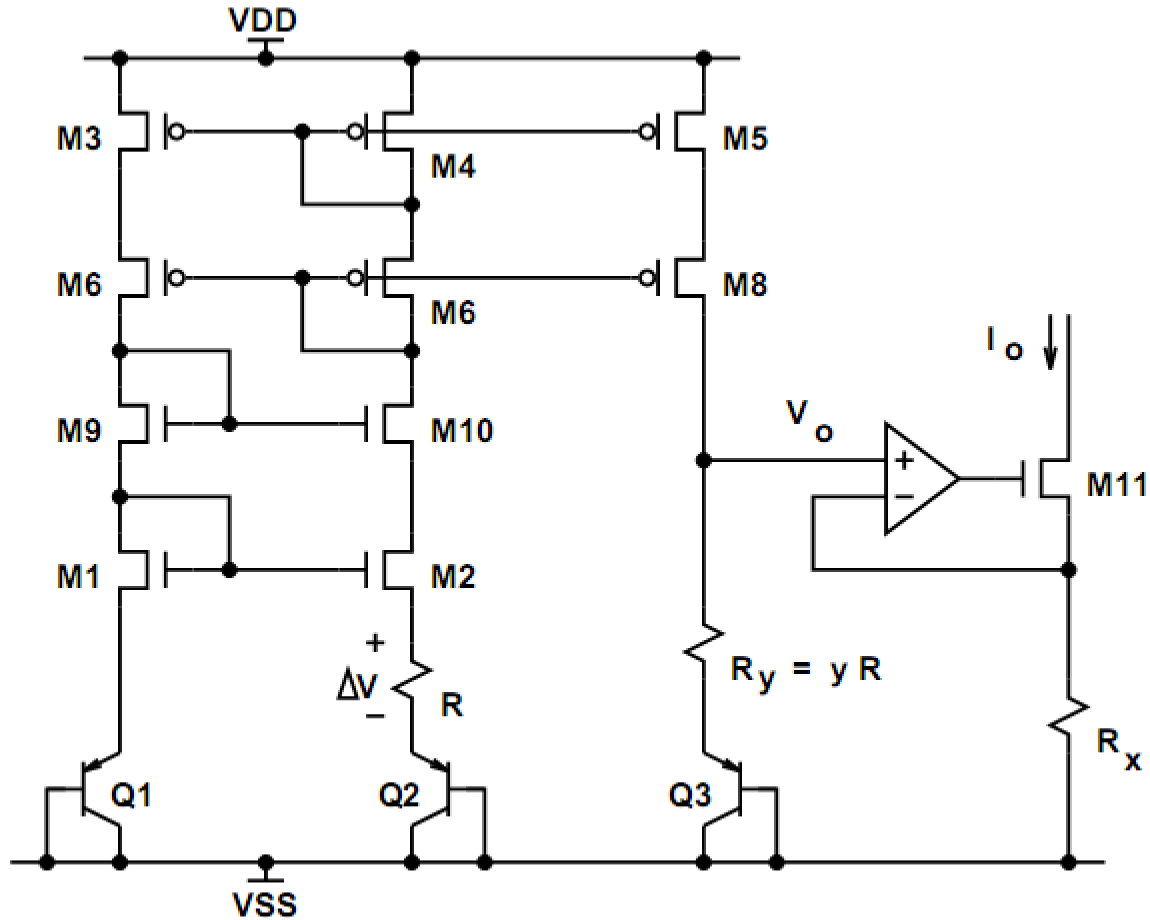
Bandgap reference



$$\frac{I_1}{I_2} = \frac{R_2}{R_1} \qquad V_{R2} = \frac{R_2}{R_3}\Delta V_{BE} \qquad \Delta V_{BE} = U_T \ln\left(\frac{I_1}{I_2}\cdot\frac{I_{S2}}{I_{S1}}\right) = U_T \ln\left(\frac{R_2}{R_1}\cdot\frac{I_{S2}}{I_{S1}}\right)$$

$$V_O = |V_{BE1}| + V_{R2} = |V_{BE1}| + \frac{R_2}{R_3}\Delta V_{BE} = |V_{BE1}| + U_T \times \frac{R_2}{R_3}\ln\left(\frac{R_2}{R_1}\cdot\frac{I_{S2}}{I_{S1}}\right)$$

Bandgap reference



Bandgap reference

Let Q2=Q3, $I_{S2}/I_{S1} = n$, and M3=M4=M5, then

$$\Delta V = U_T \ln(n)$$

The output voltage, V_O, and current, I_O, are thus

$$V_O = V_{BE3} + U_T \cdot y \ln(n) \quad \text{and} \quad I_O = \frac{V_O}{R_x}$$

- A PTAT current from M8 develops a U_T-dependent voltage across resistor R_y. A proper choice of the ratio y can give a band-gap voltage at V_O.
- All currents are proportional to T.
- If desired, a temperature-independent output current can be realized by choosing y to give an appropriate TC to V_O to cancel the TC of resistor R_2.