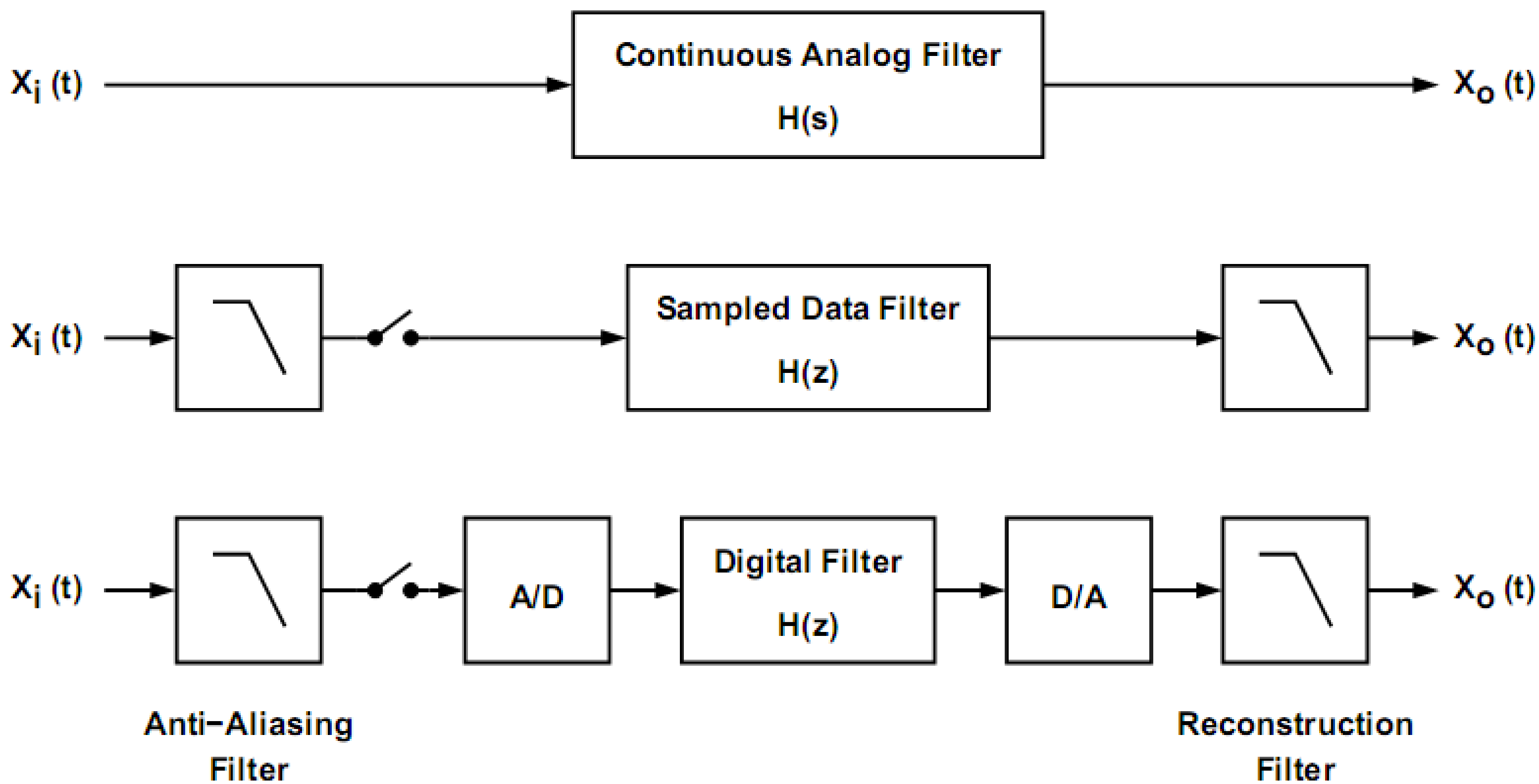
Analog Filters

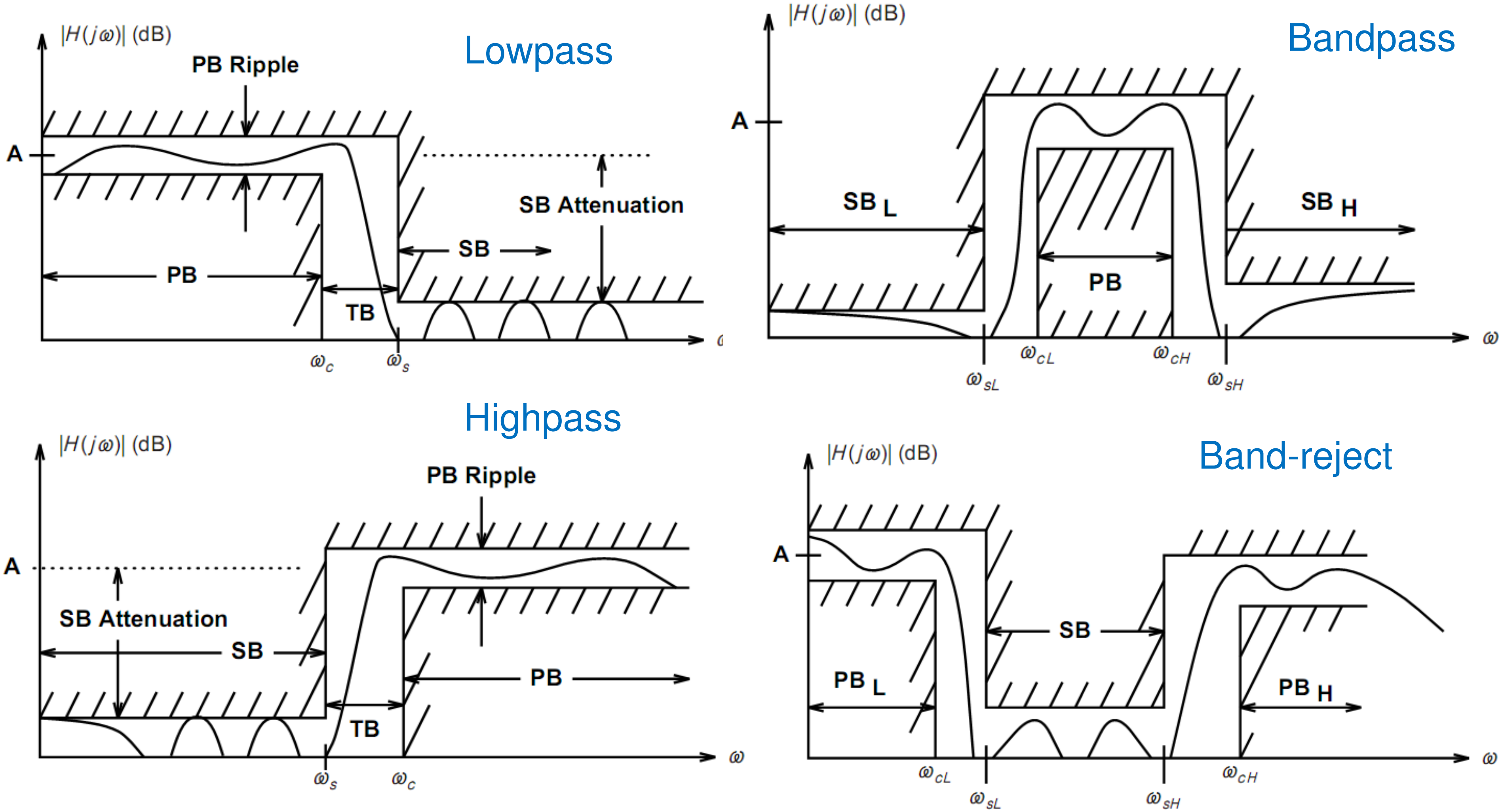
Apinunt Thanachayanont

Filters

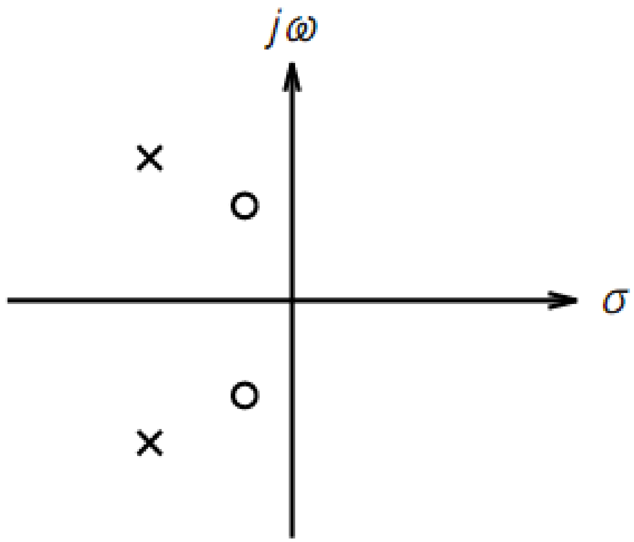


Xi (t)
Continuous Analog Filter
H(s)
Xo (t)
Xi (t)
Sampled Data Filter
H(z)
Xo (t)
Xi (t)
A/D
Digital Filter
H(z)
D/A
Xo (t)
Anti−Aliasing Filter
Reconstruction Filter

Types of filters



Second-order filters (biquadratic function)



$$H(s) = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + b_1 s + b_0}$$

$$= \frac{a_2 (s - z_1)(s - z_2)}{(s - p_1)(s - p_2)}$$

$$= K \cdot \frac{s^2 + (\omega_z / Q_z) s + \omega_z^2}{s^2 + (\omega_p / Q_p) s + \omega_p^2}$$

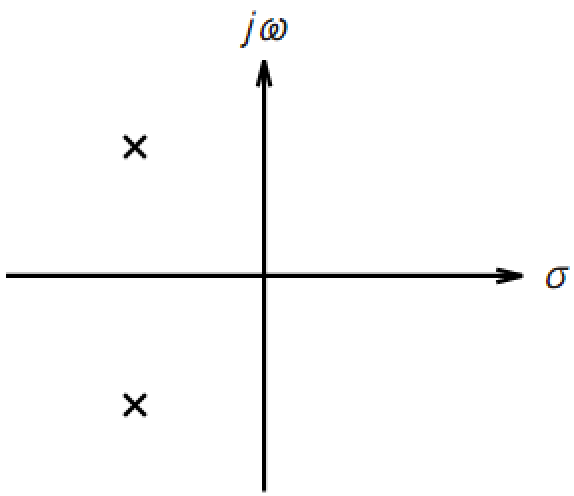
ω_p = Pole Frequency = $|p_1| = |p_2|$

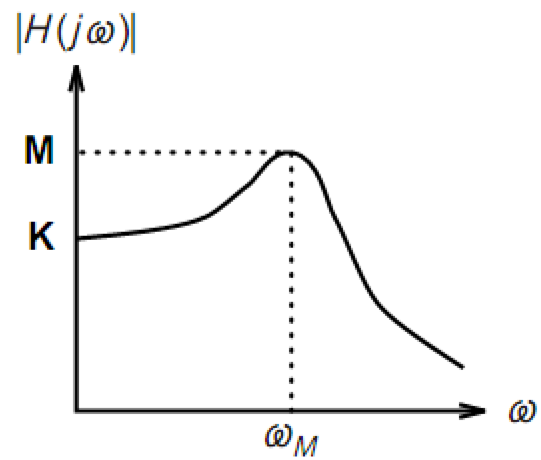
ω_z = Zero Frequency = $|z_1| = |z_2|$

Q_p = Pole Quality Factor = $\dfrac{\omega_p}{2\mathrm{Re}(p_1)}$

Q_z = Zero Quality Factor = $\dfrac{\omega_z}{2\mathrm{Re}(z_1)}$

Second-order LPF

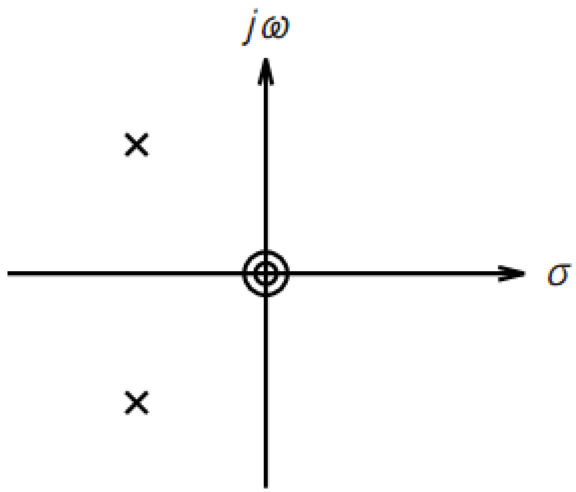


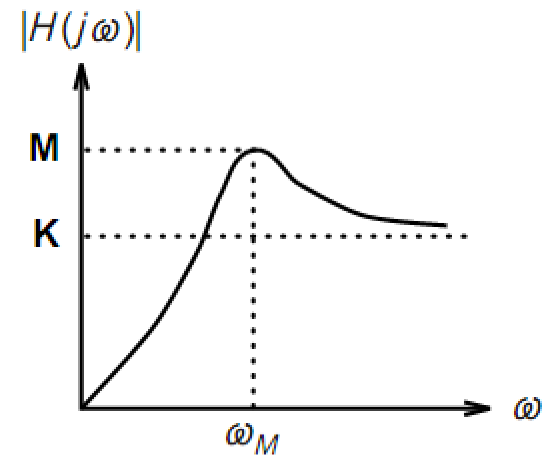


$$H(s) = \frac{K\omega_p^2}{s^2 + (\omega_p/Q_p)s + \omega_p^2}$$

$$\omega_M = \omega_p \cdot \sqrt{1 - 1/(2Q^2)} \qquad M = \frac{KQ}{\sqrt{1 - 1/(4Q^2)}}$$

Second-order HPF

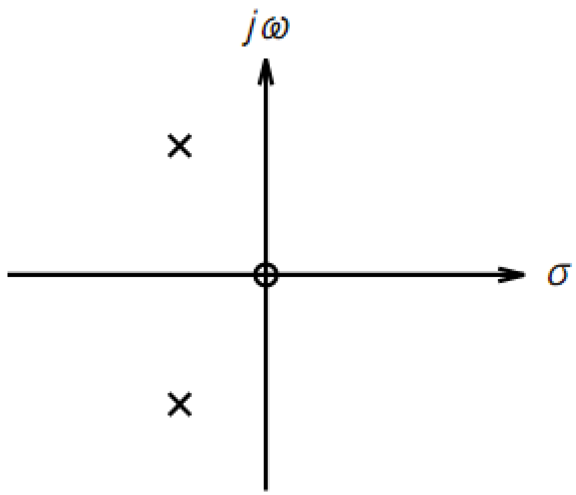


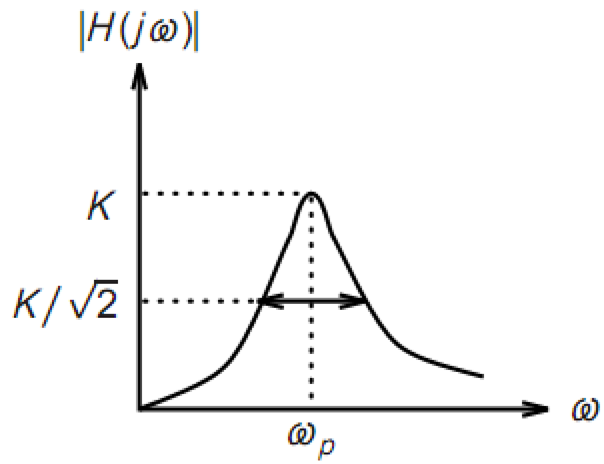


$$H(s) = \frac{K s^2}{s^2 + (\omega_p/Q_p)s + \omega_p^2}$$

$$\omega_M = \frac{\omega_p}{\sqrt{1 - 1/(2Q^2)}} \qquad M = \frac{KQ}{\sqrt{1 - 1/(4Q^2)}}$$

Second-order BPF

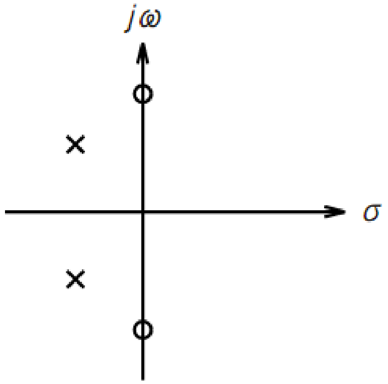


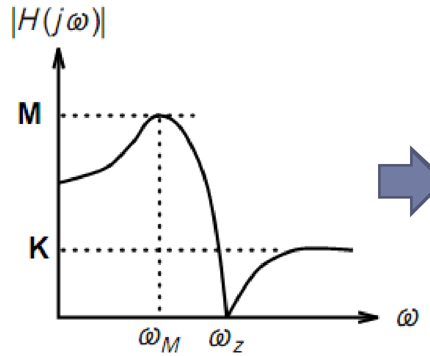


$$H(s) = \frac{K(\omega_p/Q_p)s}{s^2 + (\omega_p/Q_p)s + \omega_p^2}$$

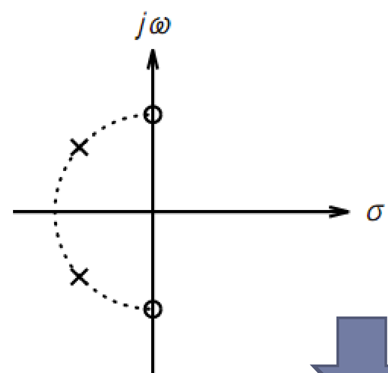
$$\text{3 dB Bandwidth} = \frac{\omega_p}{Q_p}$$

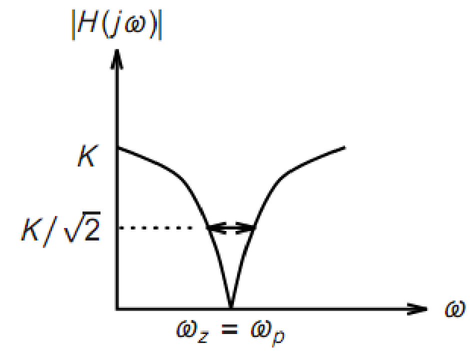
Second-order BRF





$$H(s) = \frac{K(s^2 + \omega_z^2)}{s^2 + (\omega_p/Q_p)s + \omega_p^2} \qquad \omega_z > \omega_p$$

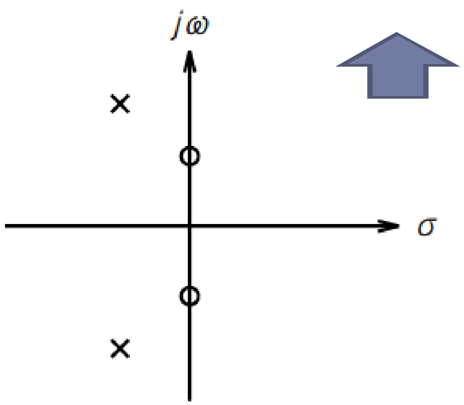


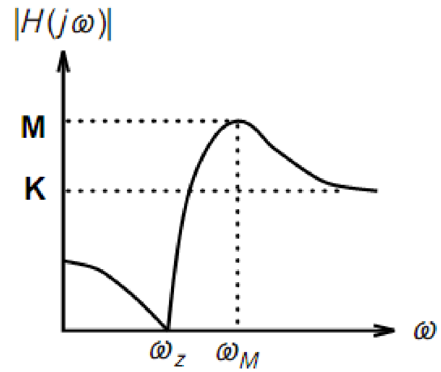


$$H(s) = \frac{K(s^2 + \omega_z^2)}{s^2 + (\omega_p/Q_p)s + \omega_p^2} \qquad \omega_z = \omega_p$$

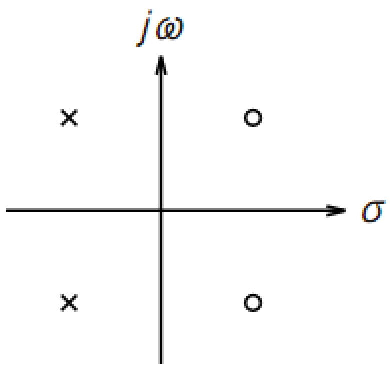
$$\text{3 dB Notch Width} = \frac{\omega_p}{Q_p}$$

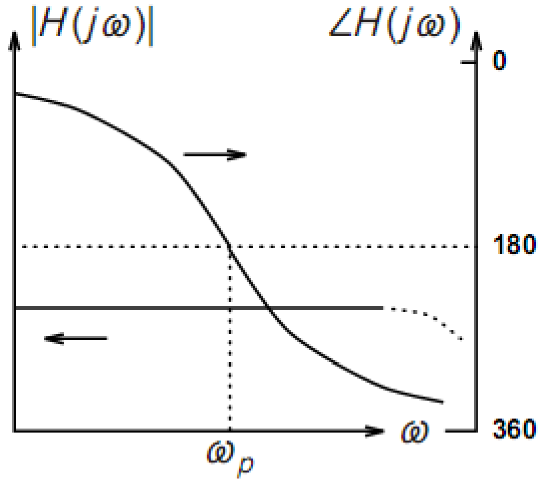
$$H(s) = \frac{K(s^2 + \omega_z^2)}{s^2 + (\omega_p/Q_p)s + \omega_p^2} \qquad \omega_z < \omega_p$$

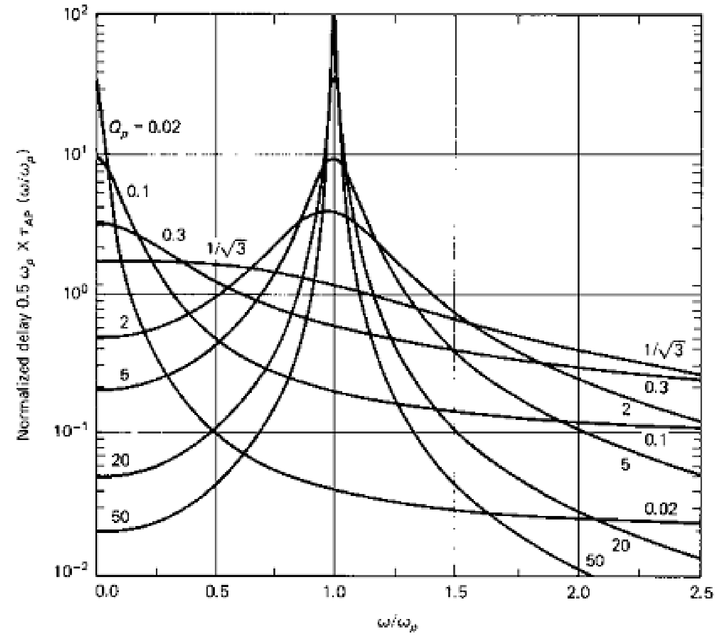




Second-order all-pass filter (APF)







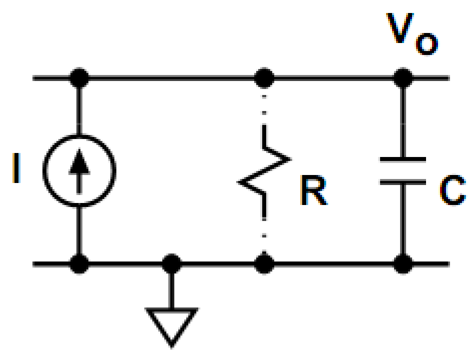
$$H(s) = K \cdot \frac{s^2 - (\omega_p/Q_p)s + \omega_p^2}{s^2 + (\omega_p/Q_p)s + \omega_p^2} \qquad |H(j\omega)| = K$$

$$\phi(\omega_n) = -2\tan^{-1}\frac{\omega_n/Q_p}{1-\omega_n^2} \qquad \omega_n = \frac{\omega}{\omega_p}$$

$$\text{Group Delay} = \tau = -\frac{d\phi(\omega)}{d\omega} \qquad \tau_n(\omega_n) = \omega_p \tau(\omega_n) = \frac{2}{Q_p} \cdot \frac{1+\omega_n^2}{(1-\omega_n^2)^2 + (\omega_n/Q_p)^2}$$

Active-RC filters

- C integrator



$$\frac{V_o}{I} = \frac{1}{j\omega C + G} = \frac{1}{j\omega C\left[1 - j\frac{G}{\omega C}\right]} = \frac{1}{j\omega C\left[1 - j\frac{1}{Q_I(\omega)}\right]}$$

$$Q_I(\omega) = \frac{\omega C}{G}$$

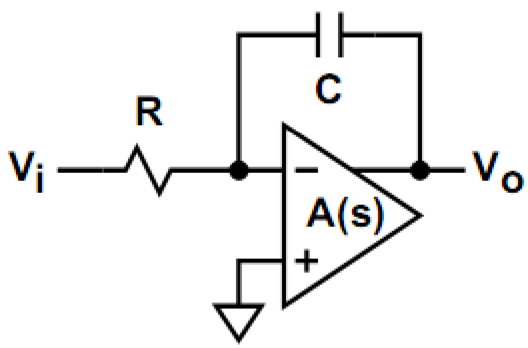
The transfer function of an integrator can be expressed as

$$H(j\omega) = \frac{1}{F(j\omega)} = \frac{1}{j\mathrm{Im}[F(j\omega)] + \mathrm{Re}[F(j\omega)]} = \frac{1}{j\omega\tau + q} = \frac{1}{j\omega\tau\left[1 - j\frac{1}{Q_I(\omega)}\right]}$$

$$Q_I(\omega) = \frac{\mathrm{Im}[F(j\omega)]}{\mathrm{Re}[F(j\omega)]} = \frac{\omega\tau}{q}$$

- Q_I is the *quality factor* of the integrator.
- For an ideal integrator, $Q_I \rightarrow \infty$ and $q \rightarrow 0$.

Active-RC inverting integrator



$$\frac{V_o}{V_i}(s) = -\frac{1}{sRC}\cdot\frac{1}{1+\frac{1}{A(s)}\left[1+\frac{1}{sRC}\right]}$$

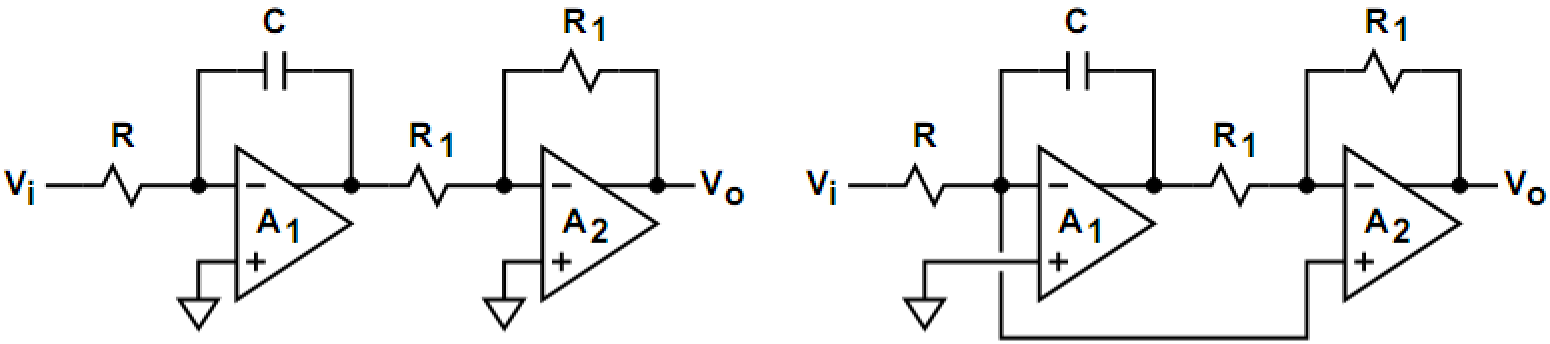
Let $A(s) = \omega_u/s$, then

$$\frac{V_o}{V_i}(s) = -\frac{1}{sRC}\cdot\frac{1}{1+s/\omega_u+1/(\omega_u RC)} \approx -\frac{1}{sRC}\cdot\frac{1}{1+s/\omega_u} \quad \text{if} \quad \omega_u \gg \frac{1}{RC}$$

$$\frac{V_o}{V_i}(j\omega) = -\frac{1}{j\omega RC-\omega^2 RC/\omega_u} = -\frac{1}{j\omega\tau+q}$$

$$\tau = RC \qquad q = -\frac{\omega^2 RC}{\omega_u} = -\frac{\omega RC}{|A(j\omega)|} \qquad Q_I = \frac{\omega\tau}{q} = -\frac{\omega_u}{\omega} = -|A(j\omega)|$$

Non-inverting integrator



Let $A_1 = A_2 = A$

$$\frac{V_o}{V_i} = \frac{1}{sRC} \cdot \frac{1}{1 + \frac{3}{A} + \frac{1}{sRCA} + \frac{2}{A^2} + \frac{2}{sRCA^2}}$$

$$Q_I = -\frac{1}{3}|A(j\omega)|$$

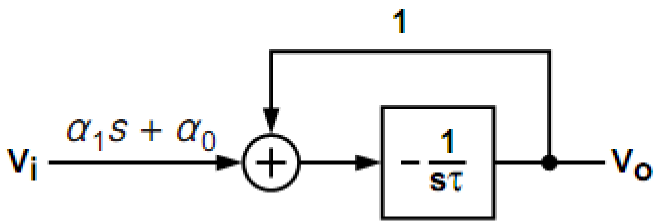
Let $A_1 = A_2 = A$

$$\frac{V_o}{V_i} = \frac{1}{sRC} \cdot \frac{1}{1 + \frac{1}{A} + \frac{1}{sRCA}}$$

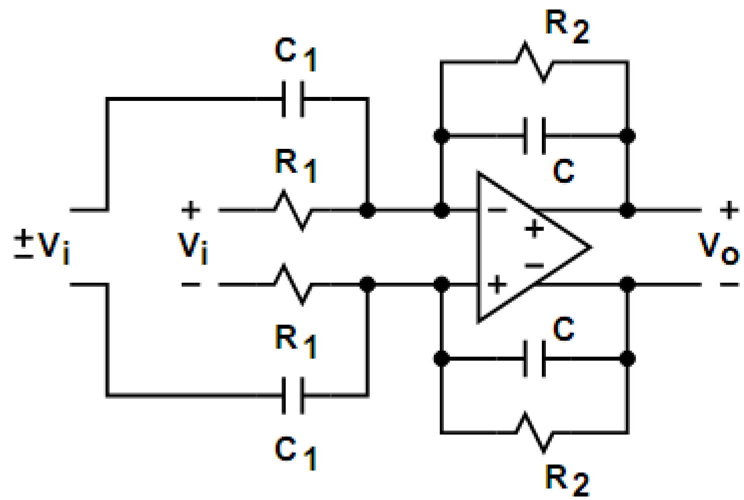
$$Q_I = -|A(j\omega)|$$

1^{st}-order filter

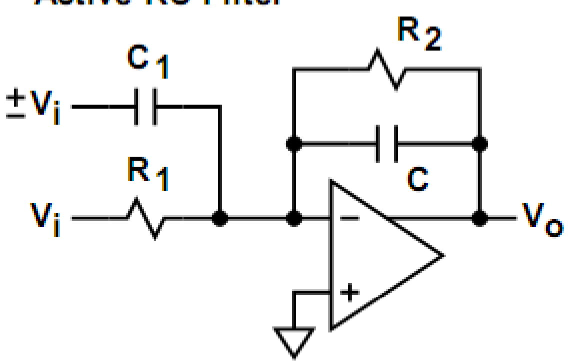
State-Variable Topology



Fully-Differential Active-RC Filter

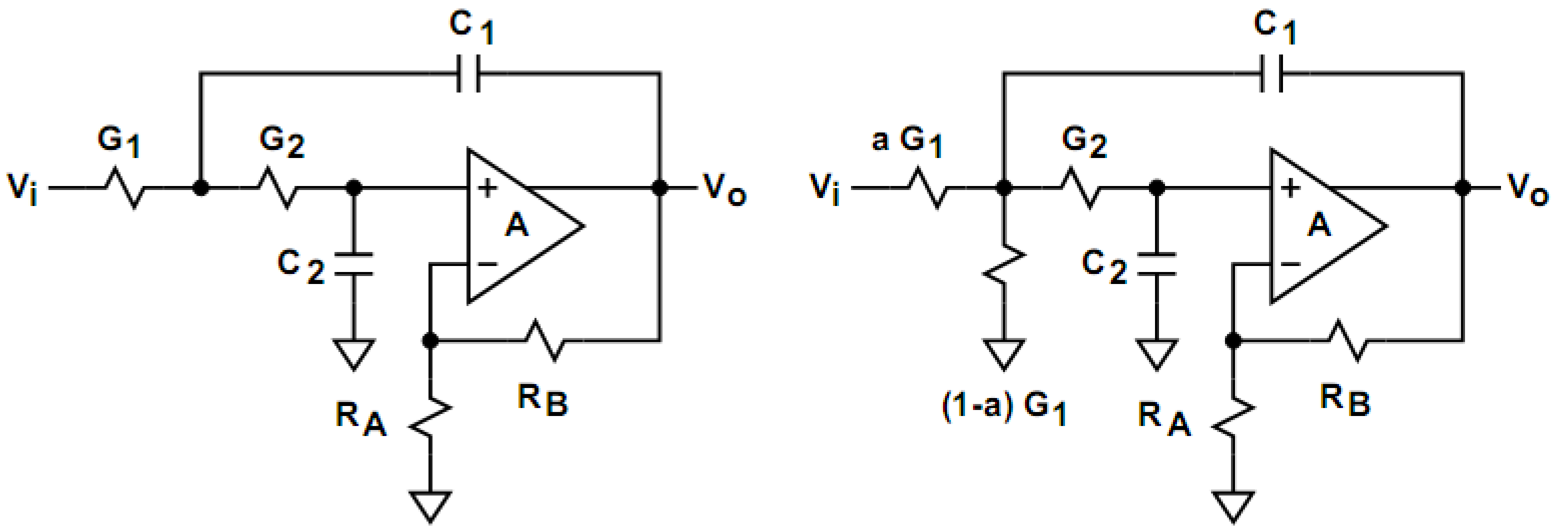


Active-RC Filter



$$\frac{V_o}{V_i} = -\frac{\alpha_1 s + \alpha_0}{s\tau + 1} = -\frac{\pm sC_1 + G_1}{sC + G_2} = -\frac{R_2}{R_1} \cdot \frac{\pm sR_1C_1 + 1}{sR_2C + 1}$$

Single amplifier 2nd-order filter – Sallen-Key LP biquad filter



$$H(s) = \frac{V_o}{V_i} = \frac{K G_1 G_2 \frac{1}{1+K/A}}{s^2 C_1 C_2 + s\left[C_2(G_1+G_2) + C_1 C_2\left(1 - K\frac{1}{1+K/A}\right)\right] + G_1 G_2}$$

$$K = a \cdot \left(1 + \frac{R_B}{R_A}\right) \qquad a \le 1$$

Single amplifier 2nd-order filter – Sallen-Key LP biquad filter

Let $A = \infty$ and $C_1 = C_2 = C$, then

$$H(s) = \frac{KG_1G_2/C^2}{s^2 + s[G_1 + G_2(2-k)]/C + G_1G_2/C^2} = K \cdot \frac{\omega_p^2}{s^2 + s\omega_p/Q + \omega_p^2}$$

$$\omega_p^2 = \frac{G_1G_2}{C^2} \qquad Q = \frac{\sqrt{G_1G_2}}{G_1 + G_2(2-K)} \qquad K = a \cdot \left(1 + \frac{R_B}{R_A}\right)$$

If $a = 1$, $R_1 = R_2 = R$, we have

$$\omega_p = \frac{1}{RC} \qquad Q = \frac{1}{3-K} \qquad S_K^Q = 3Q - 1$$

- Minimal use of opamp, at the expense of more passive components.
- Sensitive to parasitic capacitors.
- Widely used to realize the on-chip anti-aliasing and reconstruction filters.

Single amplifier 2nd-order filter – Sallen-Key LP biquad filter

Let $a = 1$, $R_1 = R_2 = R$, $C_1 = C_2 = C$, $A = \omega_u/s$,

$$H'(s) = K \cdot \frac{\omega_p^2 \frac{1}{1+K/A}}{s^2 + s\omega_p\left(3 - K\frac{1}{1+K/A}\right) + \omega_p^2} \approx K \cdot \frac{\omega_p^2(1 - K/A)}{s^2 + s\omega_p\left[3 - K(1 - K/A)\right] + \omega_p^2}$$

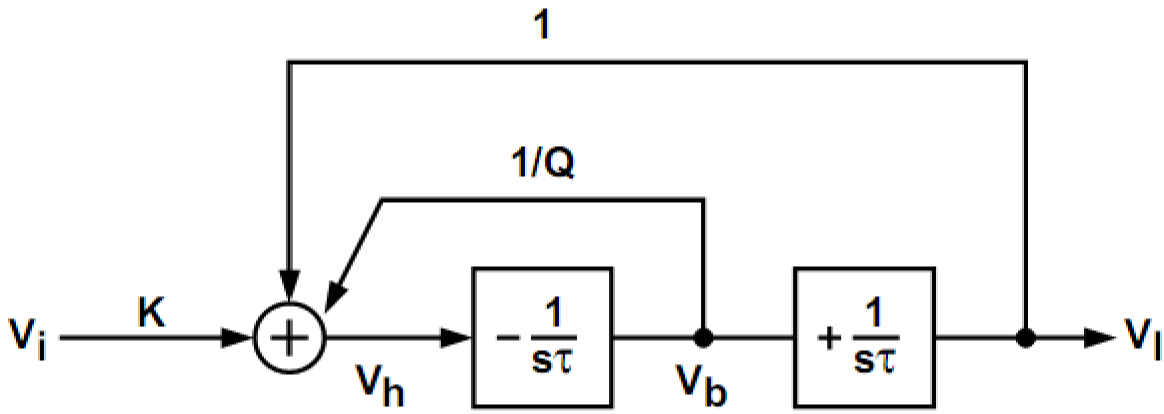
$$\Rightarrow \qquad H'(s) \approx K \cdot \frac{\omega_p^2(1 - sK/\omega_u)}{s^2(1+\epsilon) + s\omega_p(3 - K) + \omega_p^2} = K \cdot \frac{\omega_p'^2(1 - K/\omega_u)}{s^2 + s\omega_p'/Q' + \omega_p'^2}$$

$$\omega_p' = \frac{\omega_p}{\sqrt{1+\epsilon}} \approx \omega_p\left(1 - \frac{\epsilon}{2}\right) = \omega_p - \Delta\omega_p \qquad Q' = Q\sqrt{1+\epsilon} \approx Q\left(1 + \frac{\epsilon}{2}\right) = Q + \Delta Q$$

$$\epsilon = \frac{\omega_p}{\omega_u}K^2 = \frac{K^2}{|A(j\omega_p)|}$$

- $H'(s)$ has an additional positive zero at ω_u/K.
- The Sallen-Key biquad is a good low-Q LP filter with small ω_u-caused deviations.

State-variable 2nd-order filters



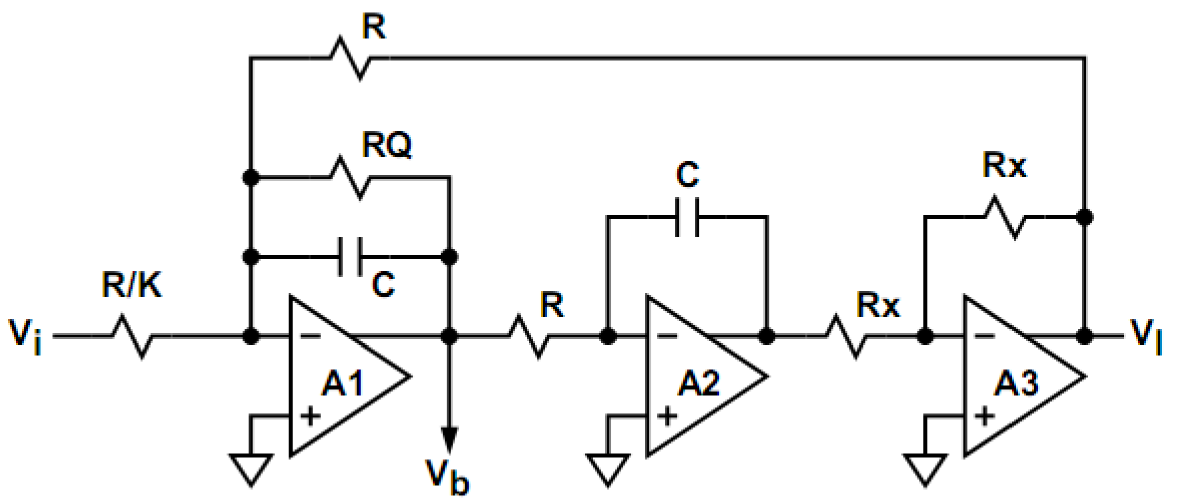
$$\frac{V_h}{V_i} = +K \cdot \frac{s^2}{s^2 + s/(Q\tau) + 1/\tau^2} = K \cdot \frac{s^2}{s^2 + s\omega_p/Q + \omega_p^2}$$

$$\frac{V_b}{V_i} = -K \cdot \frac{s/\tau}{s^2 + s/(Q\tau) + 1/\tau^2} = -K \cdot \frac{s\omega_p}{s^2 + s\omega_p/Q + \omega_p^2}$$

$$\omega_p = \frac{1}{\sqrt{\tau_1\tau_2}} = \frac{1}{\tau}$$

$$\frac{V_l}{V_i} = -K \cdot \frac{1/\tau^2}{s^2 + s/(Q\tau) + 1/\tau^2} = -K \cdot \frac{\omega_p^2}{s^2 + s\omega_p/Q + \omega_p^2}$$

Tow-Thomas (TT) biquad

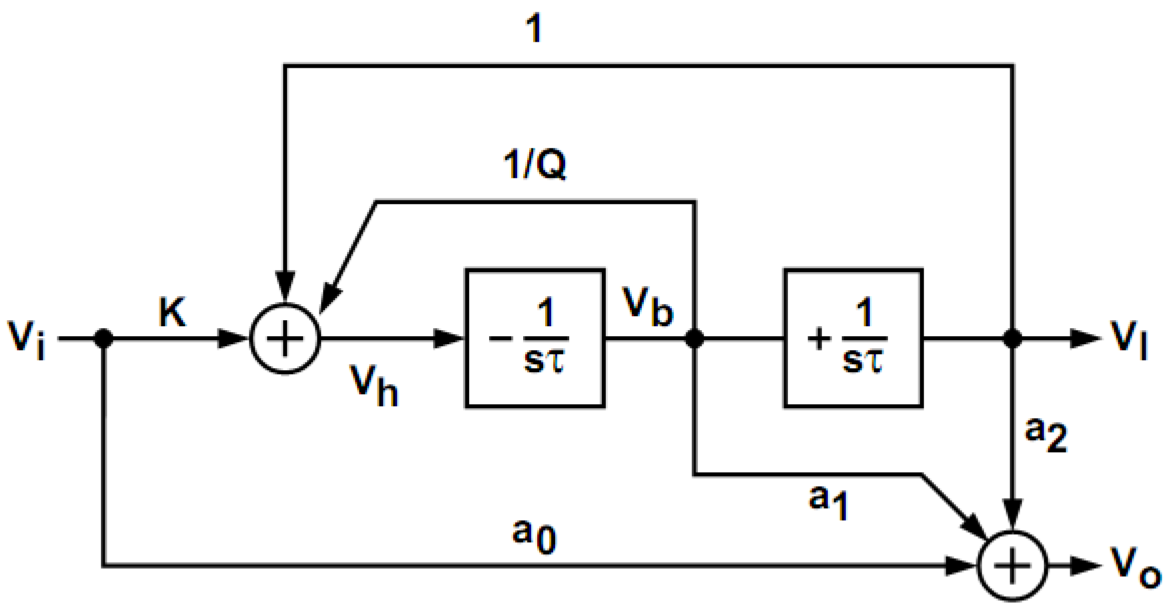


$$\frac{V_b}{V_i} = -K \cdot \frac{\omega_p s}{s^2 + s\omega_p/Q + \omega_p^2} \qquad \frac{V_l}{V_i} = -K \cdot \frac{\omega_p^2}{s^2 + s\omega_p/Q + \omega_p^2} \qquad \omega = \frac{1}{RC}$$

The sensitivities for any passive component x are

$$S_x^{\omega_p} = -1/2 \qquad \left|S_x^Q\right| \leq 1$$

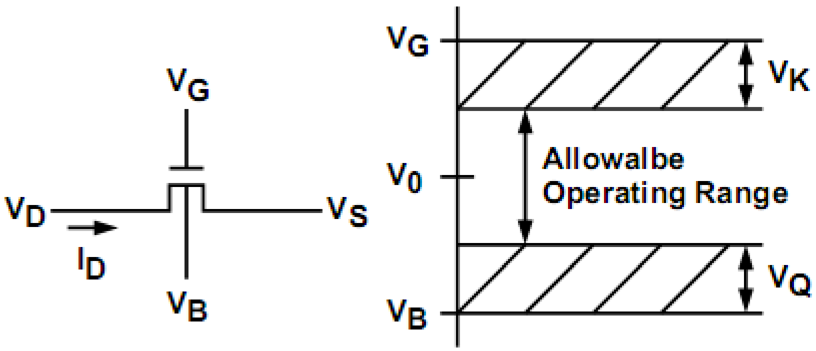
Arbitrary transmission zeros by summing



$$\frac{V_o}{V_i} = a_0 + \frac{-a_1 \cdot Ks\omega_p - a_2 \cdot K\omega_p^2}{s^2 + s\omega_p/Q + \omega_p^2} = \frac{a_0 s^2 + s(\omega_p/Q)[a_0 - a_1(KQ)] + \omega_p^2[a_0 - a_2 K]}{s^2 + s\omega_p/Q + \omega_p^2}$$

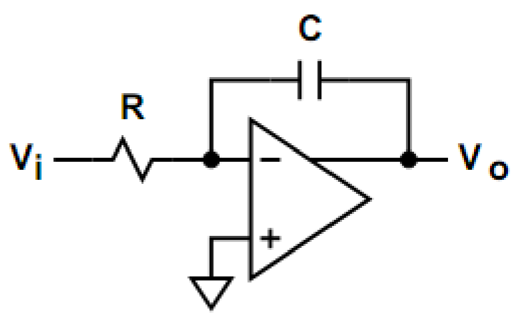
MOSFET-C filters

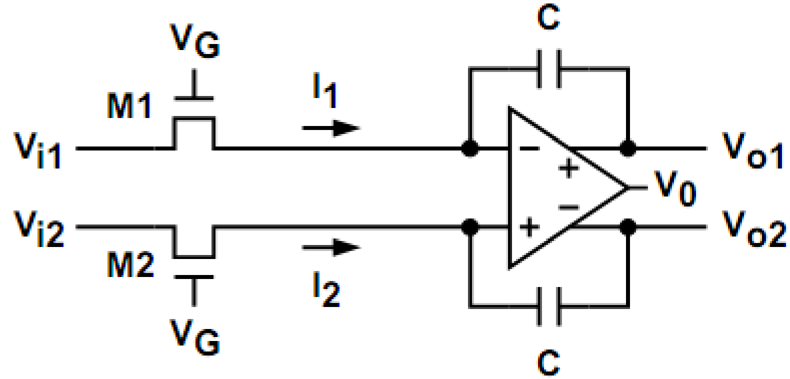
- MOSFET in the Triode region



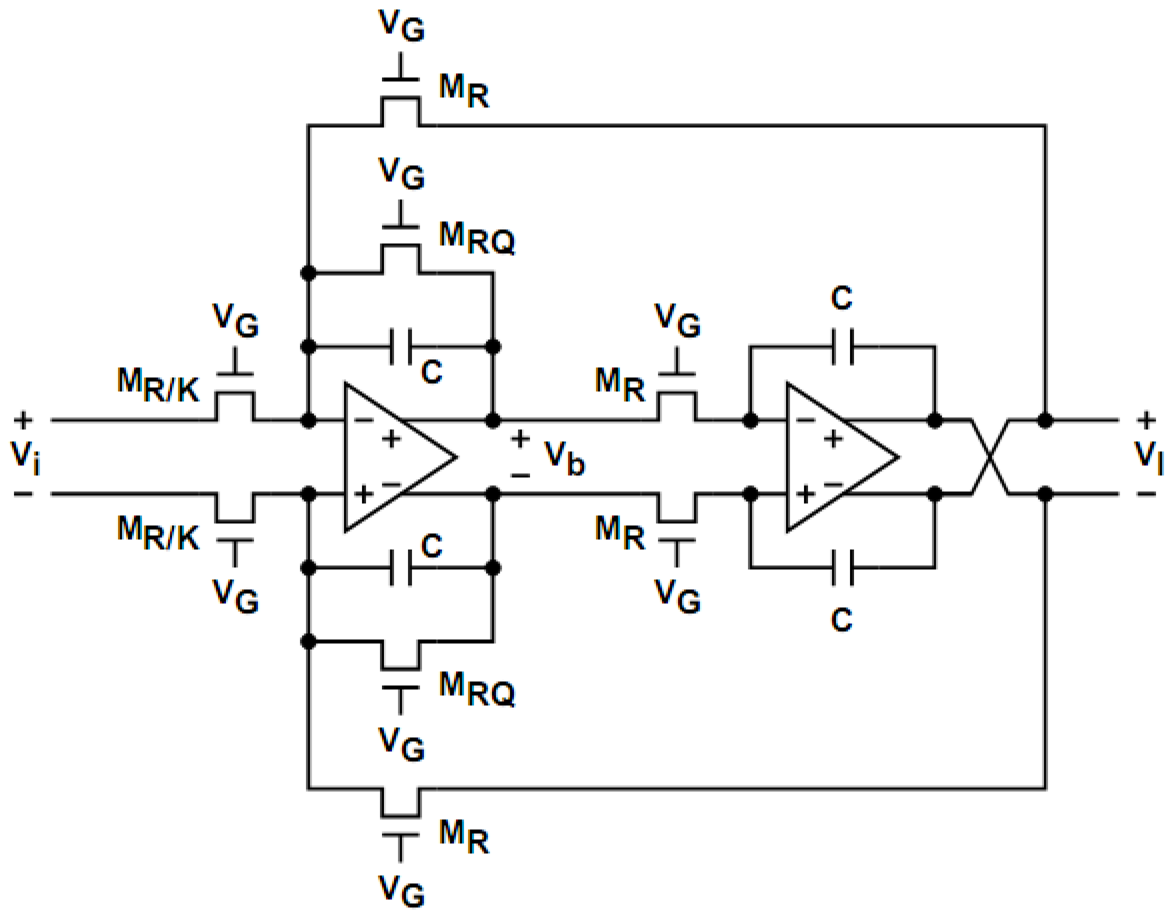
$$G = \frac{I_D}{V_{DS}} = k(V_{G0} - V_T) = \mu C_{ox}\frac{W}{L}(V_{G0} - V_T)$$

- MOSFET-C integrators



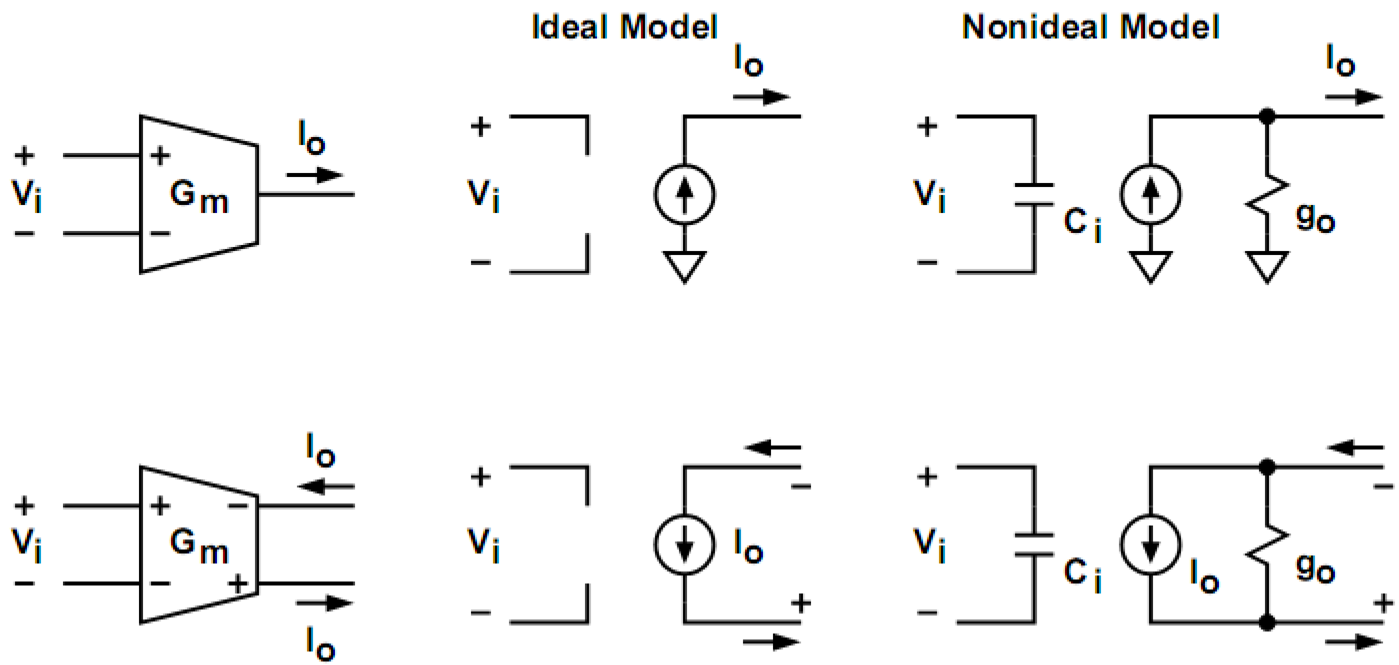


MOSFET-C TT biquad filter



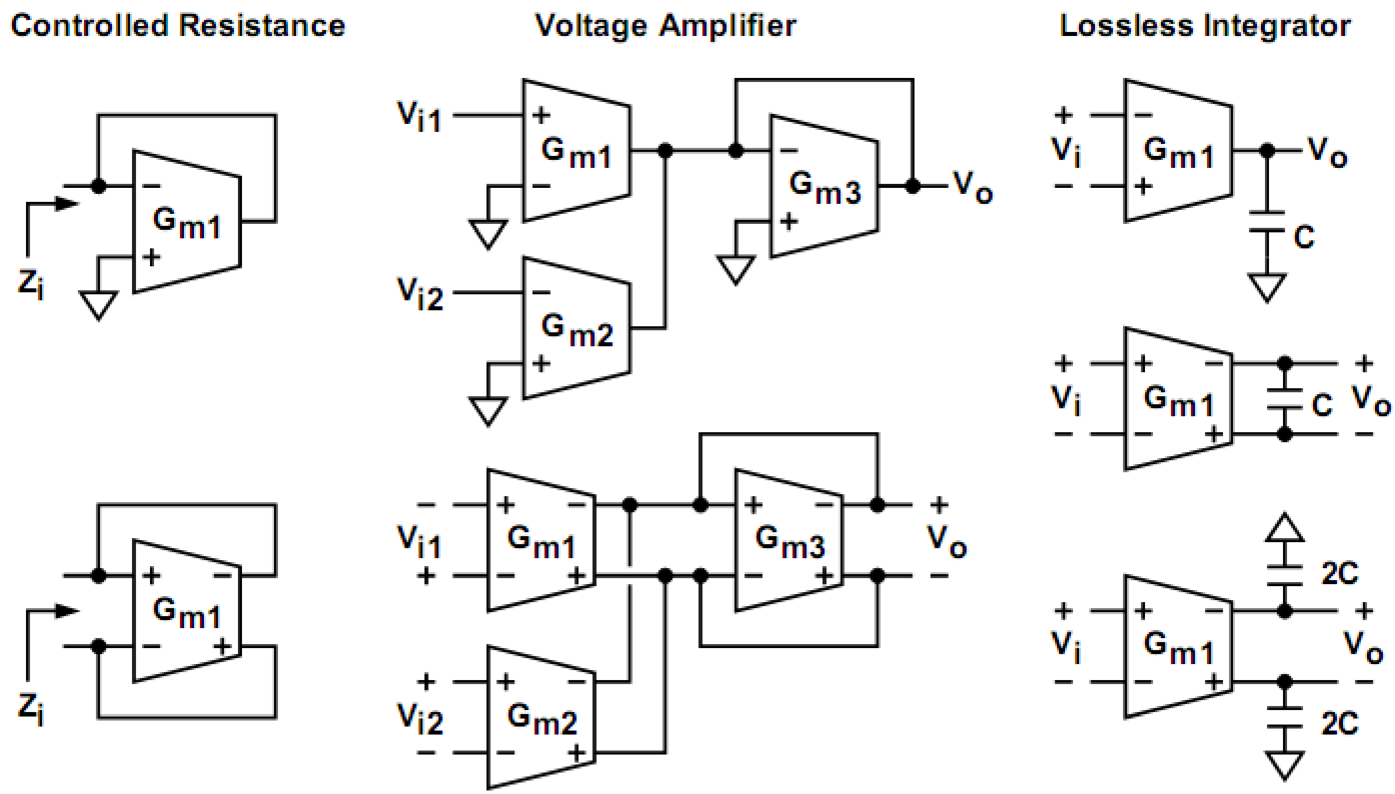
Transconductor-C filters

- Transconductor



$$I_o = G_m \times V_i$$

Transconductor basic circuits

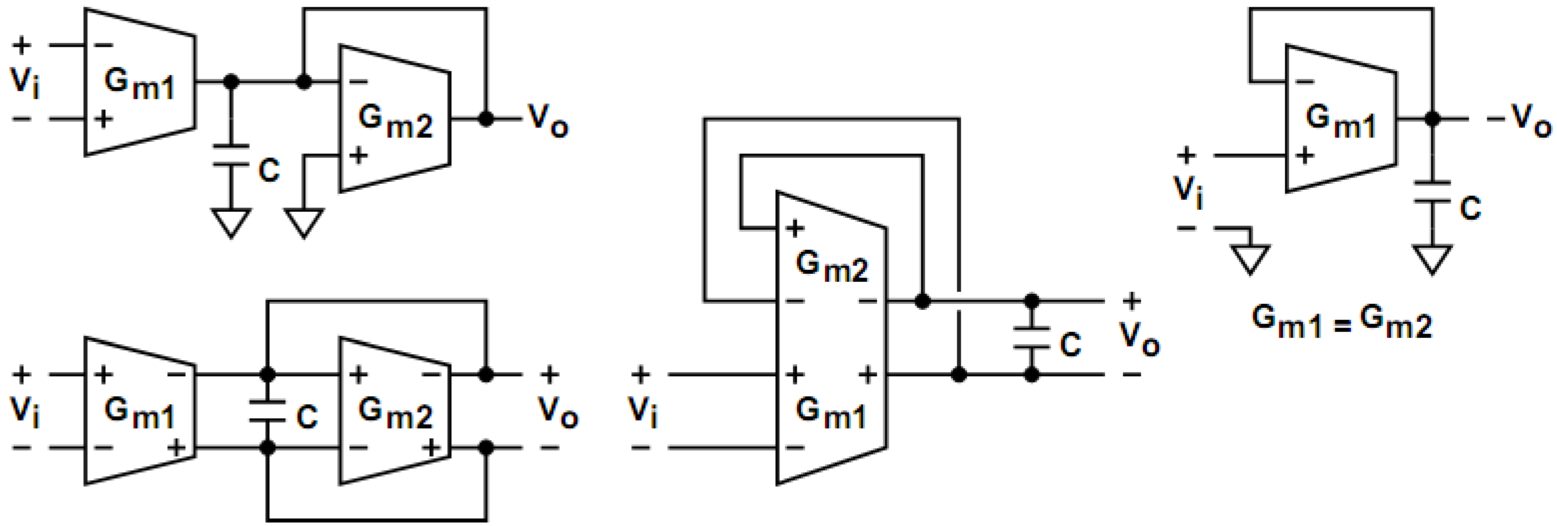


$$Z_i = \frac{1}{G_{m1}}$$

$$V_o = \frac{1}{G_{m3}} \cdot (G_{m1}V_{i1} - G_{m2}V_{i2})$$

$$\frac{V_o(s)}{V_i(s)} = -\frac{G_m}{sC}$$

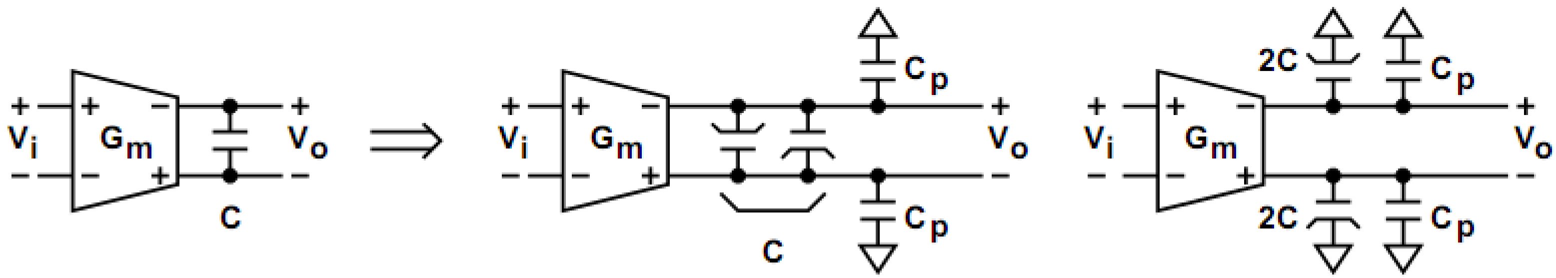
Gm-C lossy integrator



$$\frac{V_o(s)}{V_i(s)} = -\frac{G_{m1}}{sC + G_{m2}}$$

- Since no feedback for the integrators, they can be wide-band.
- A transconductor's output current should be linearly related to the input over the entire input voltage range.

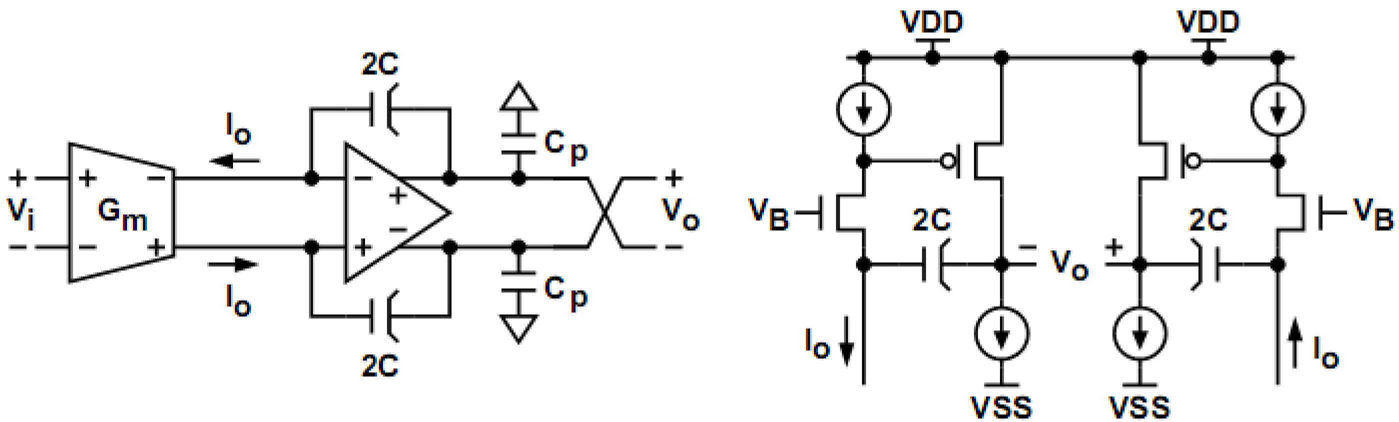
Fully-differential Gm-C integrator



$$\frac{V_o(s)}{V_i(s)} = -\frac{G_m}{s(C + C_p/2)}$$

- Can use only grounded capacitors.
- The C_p can affect the integration time constant.
- Partially nonlinear C_p can also cause linearity problems.

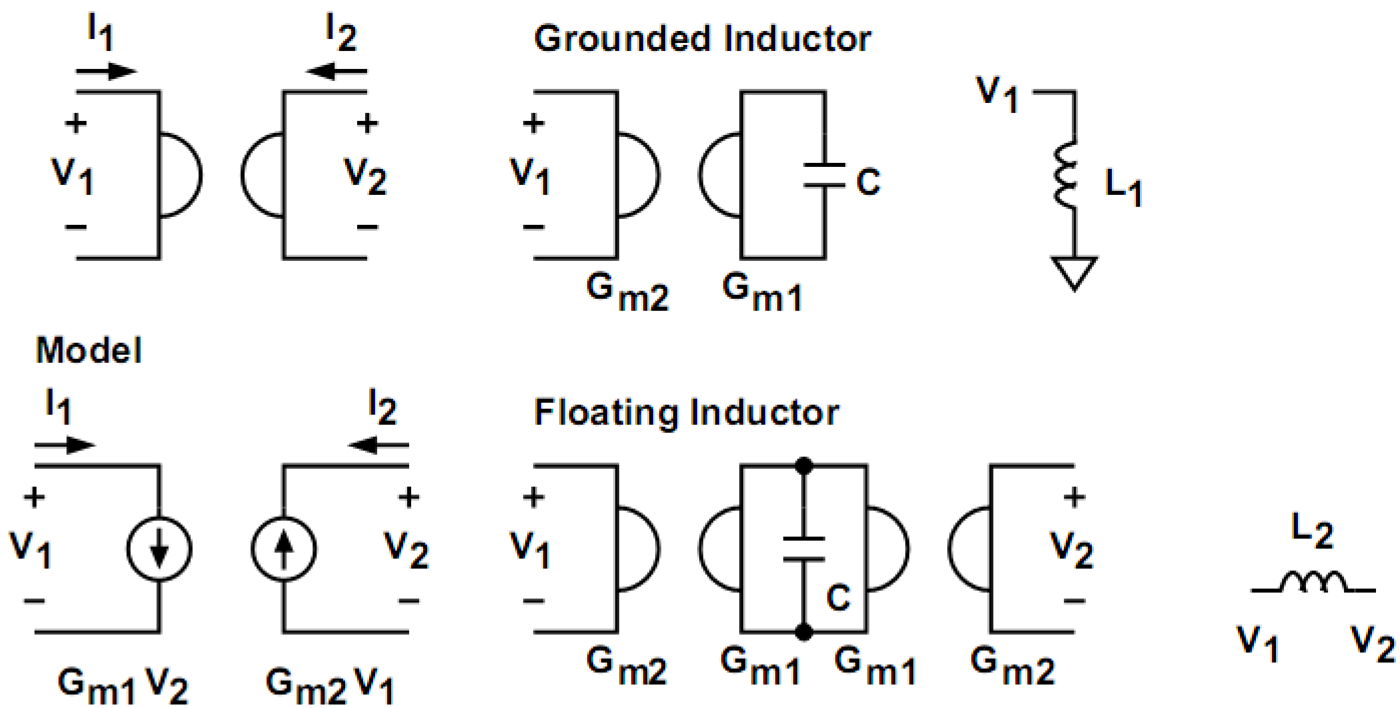
Gm-C-opamp integrator



$$\frac{V_o(s)}{V_i(s)} = -\frac{G_m}{sC}$$

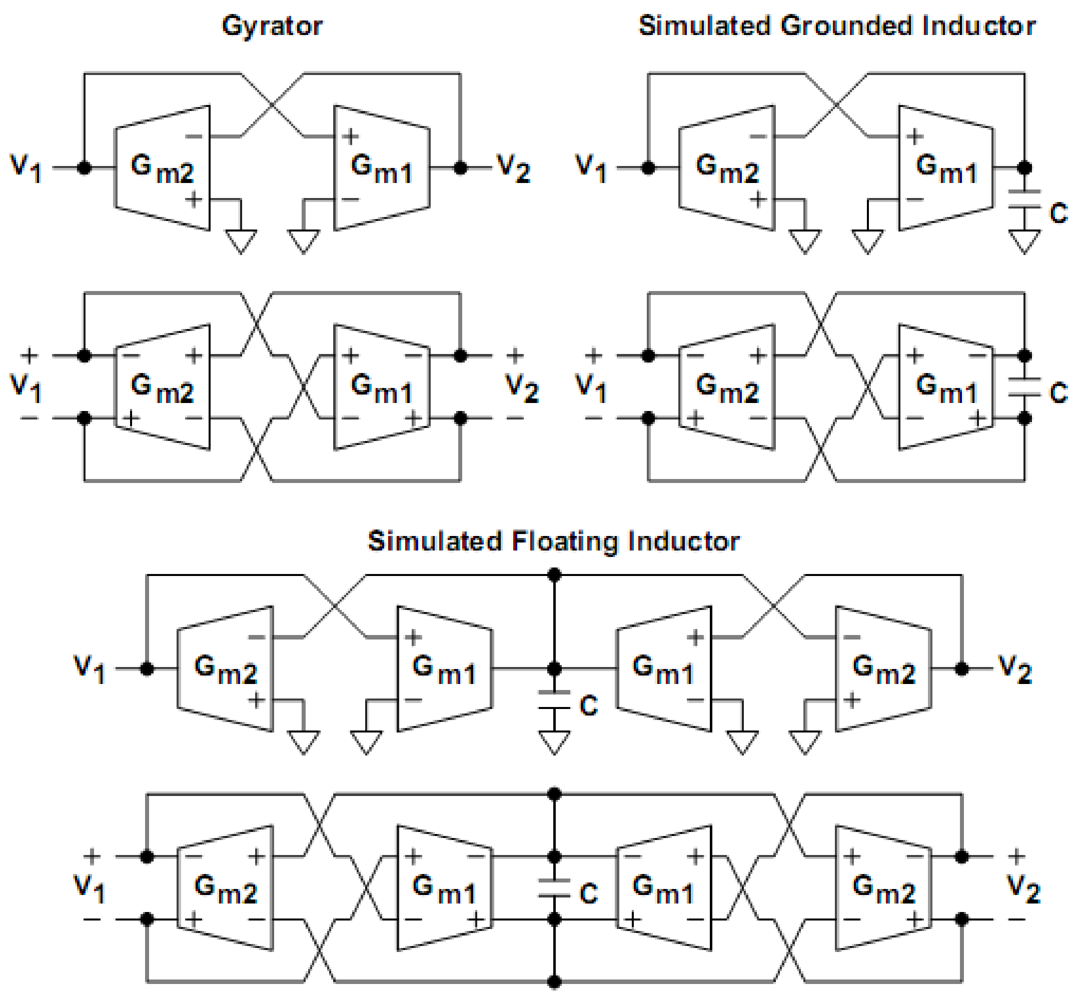
- The effects of parasitic capacitances are reduced.
- The G_m's output stage can be simplified, since no large voltage swing is required.
- The lower impedances at the G_m's output nodes make those nodes less sensitive to capacitive coupling of noise.

Gyrators

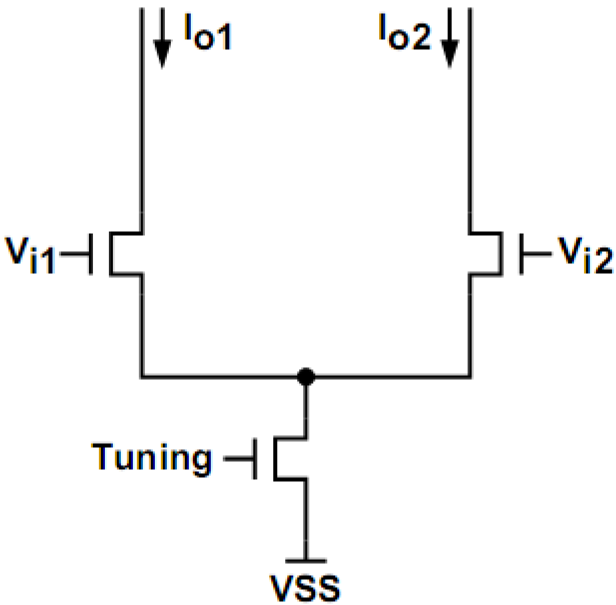


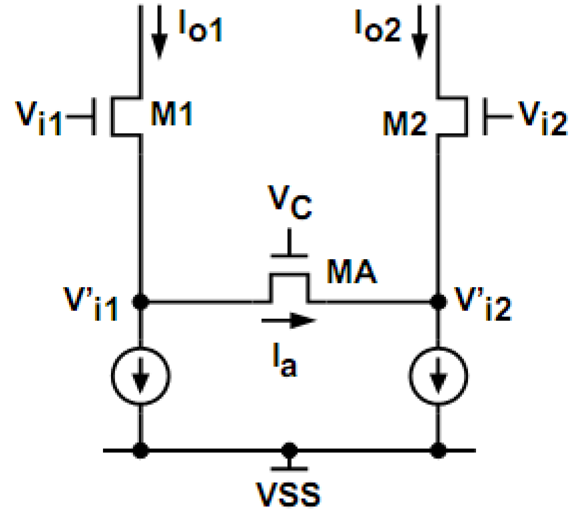
$$L_1 = \frac{C}{G_{m1} \cdot G_{m2}} \qquad L_2 = \frac{C}{G_{m1} \cdot G_{m2}}$$

Gm-C gyrators



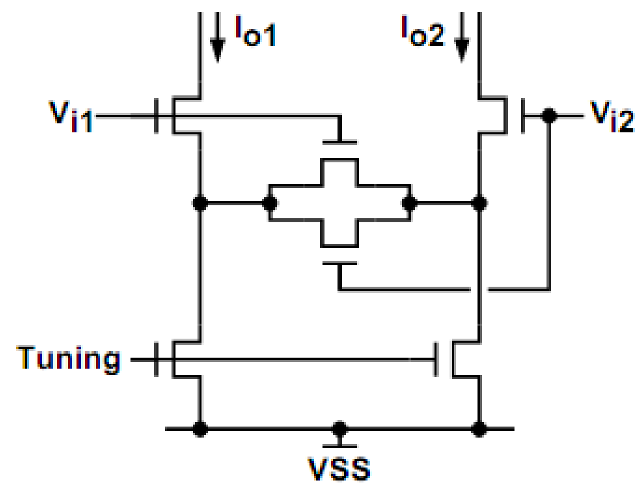
CMOS transconductors



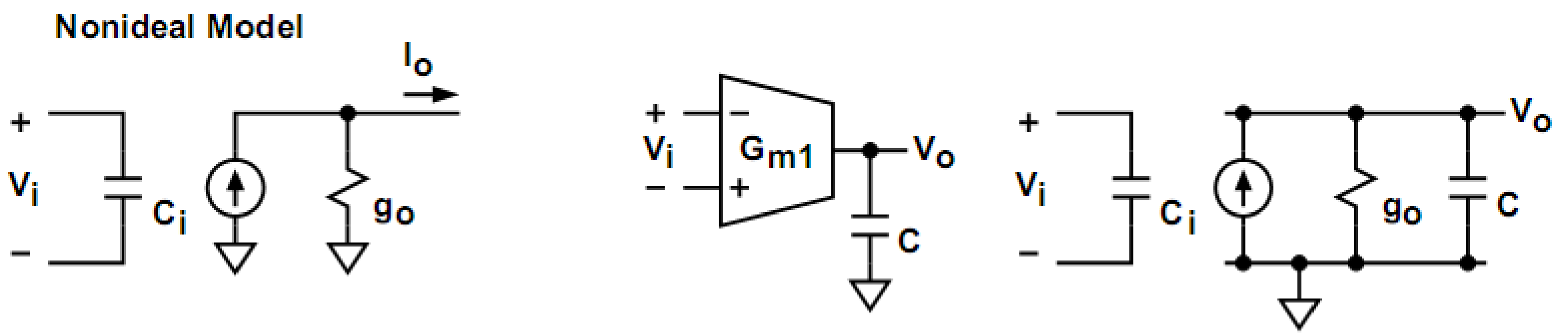


Fully Balanced Type

Adaptive Source Degeneration



Transconductor's non-idealities

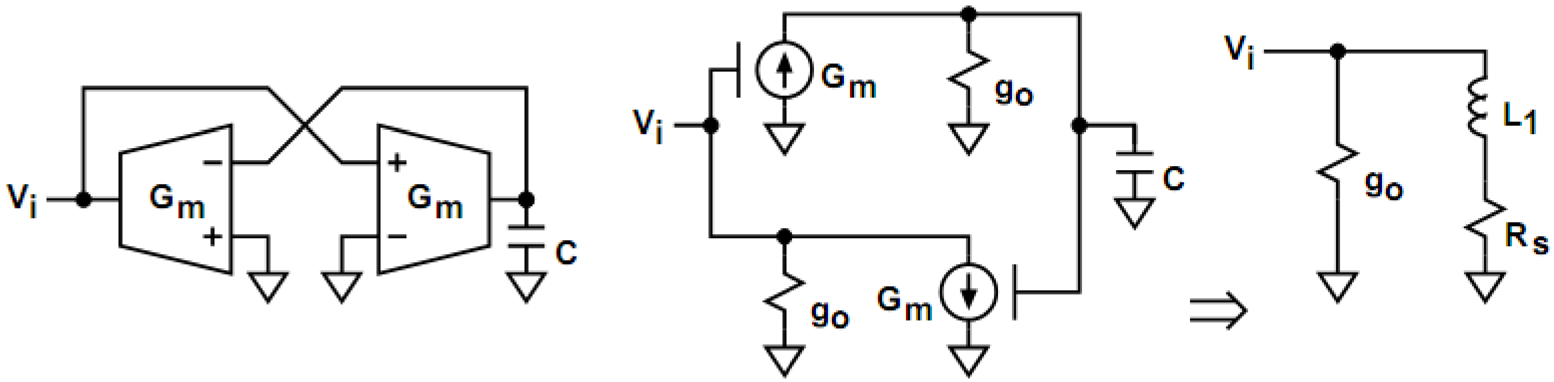


$$I_o = G_m(s) \times V_i \qquad G_m(j\omega) = \frac{G_m}{1 + j\omega/\omega_2} \approx G_m e^{-j\phi} \qquad \phi = \tan^{-1}\frac{\omega}{\omega_2}$$

For the G_m-C integrator

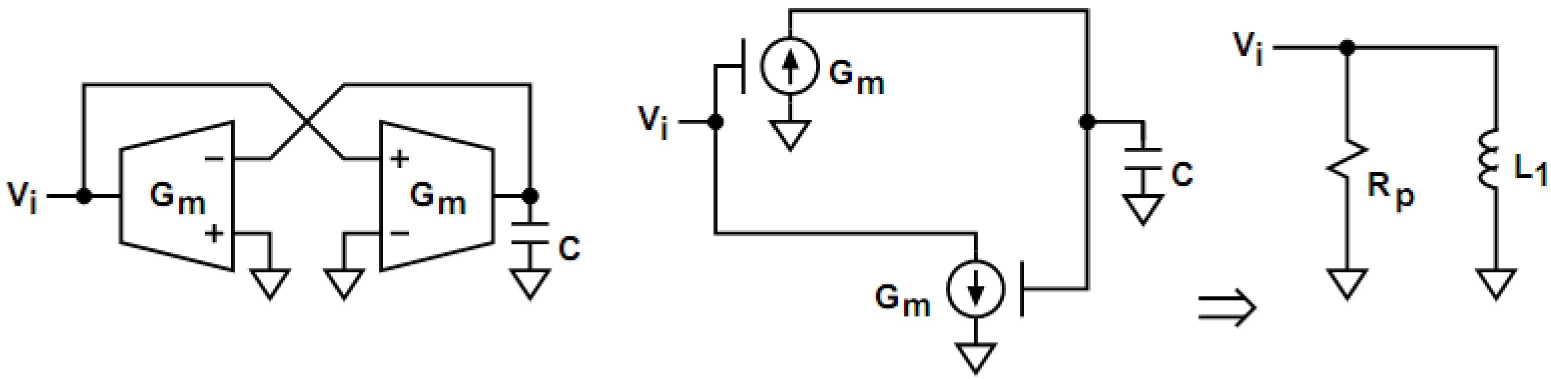
$$\frac{V_o}{V_i} = \frac{G_m}{1 + s/\omega_2} \times \frac{1}{sC + g_o} = \frac{G_m}{sC\left(1 + \frac{\omega_o}{\omega_2}\right) + g_o\left(1 + \frac{s^2}{\omega_o\omega_2}\right)} \qquad \omega_o = \frac{g_o}{C}$$

Effects of non-zero g_o on gyrator



$$L = \frac{C}{G_m^2} \qquad R_s = \frac{g_o}{G_m^2}$$

Effect of phase shift on gyrator



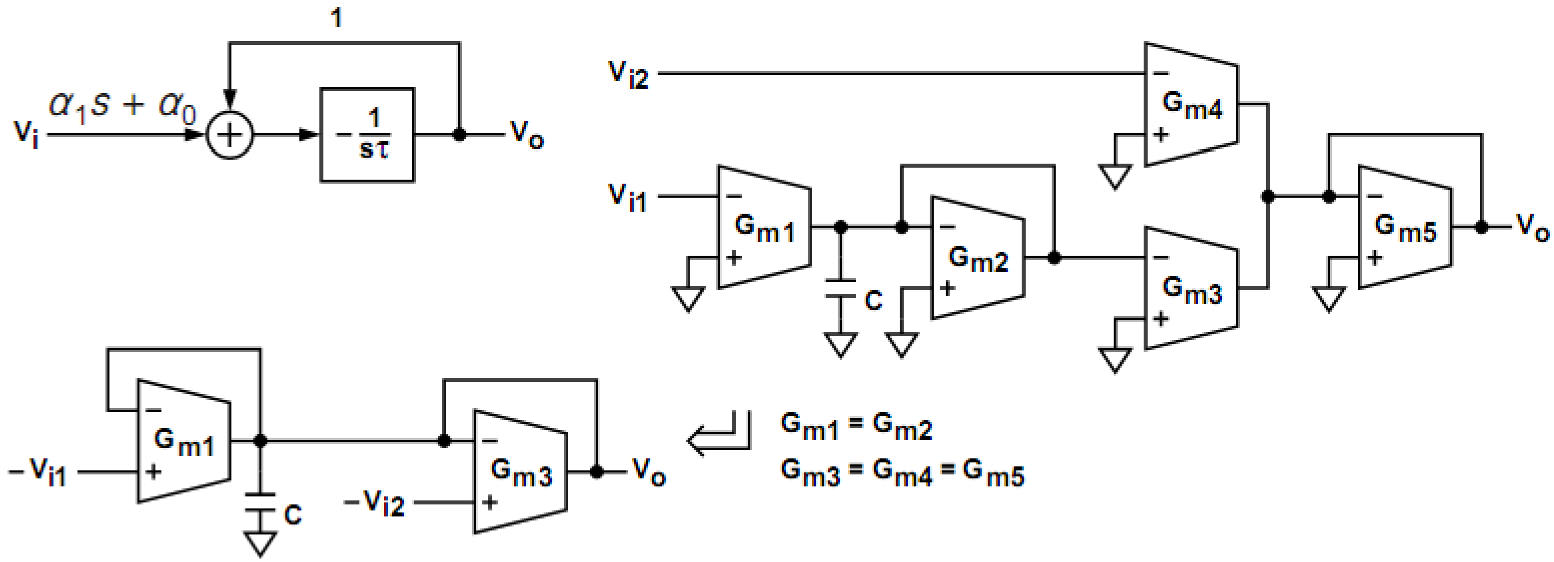
If

$$G_m(j\omega) = G_m e^{-j\phi} \qquad \phi = \tan^{-1}\left(\frac{\omega}{\omega_2}\right) \approx \frac{\omega}{\omega_2} \ll 1$$

We have

$$L = \frac{C}{G_m^2} \qquad \frac{1}{R_p} \approx -\frac{2G_m^2}{\omega C}\cdot\phi \approx -\frac{2G_m^2}{\omega_2 C} = -\frac{2}{\omega_2 L}$$

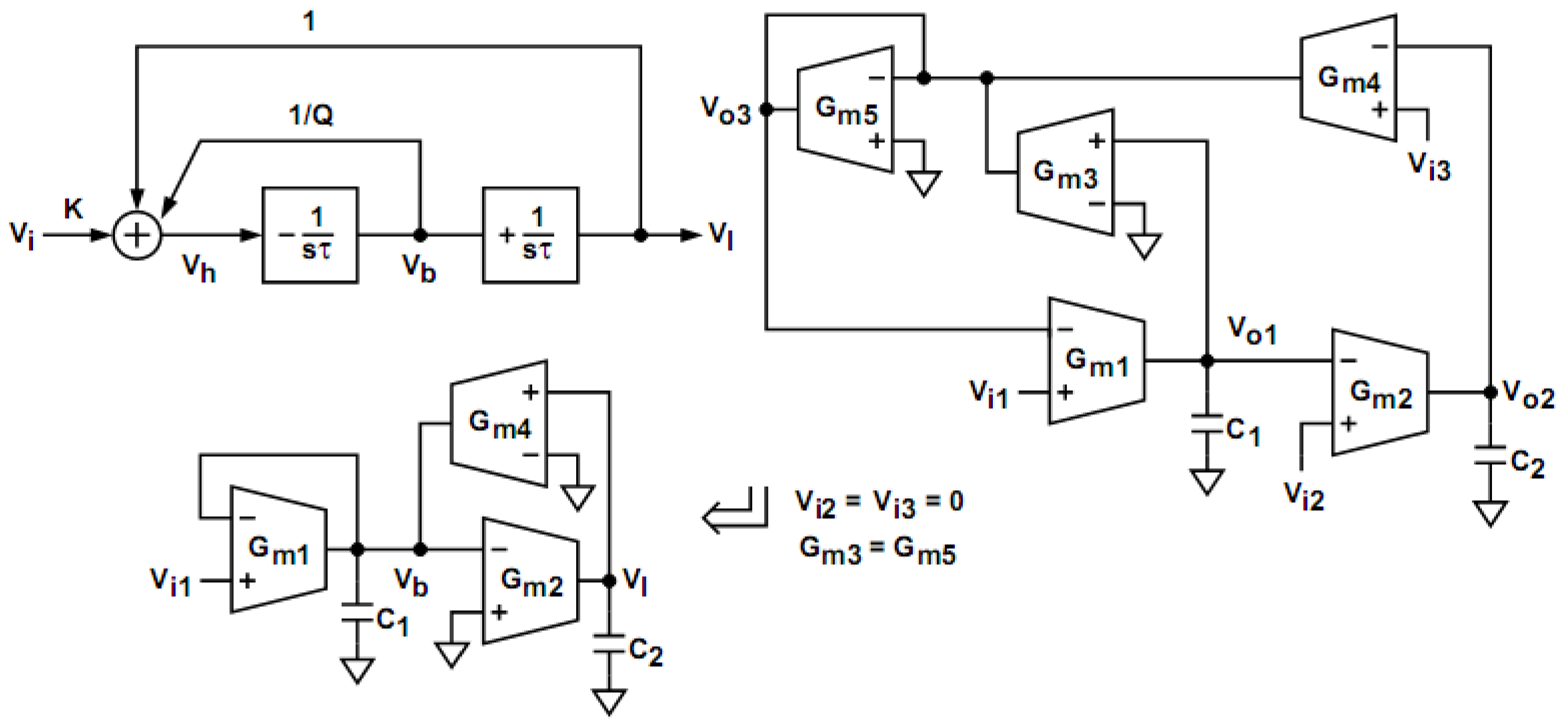
Gm-C 1st-order filter



$$H(s) = -\left(\frac{G_{m1}V_{i1}}{sC + G_{m2}} \cdot G_{m3} + G_{m4}V_{i2}\right) \cdot \frac{1}{G_{m5}} = -\frac{sCG_{m4}V_{i2} + (G_{m1}G_{m3}V_{i1} + G_{m2}G_{m4}V_{i2})}{(sC + G_{m2}) \cdot G_{m5}}$$

- The output requires another buffer to prevent loading effects.
- Use only grounded capacitors.

Gm-C 2nd-order filters



$$V_b = \frac{sC_2G_{m1}}{s^2C_1C_2 + sC_2G_{m1} + G_{m2}G_{m4}} \qquad V_l = -\frac{G_{m1}G_{m2}}{s^2C_1C_2 + sC_2G_{m1} + G_{m2}G_{m4}}$$

The transfer functions are

$$V_{o1} = [1/D(s)] \cdot [sC_2G_{m1}(G_{m5}V_{i1} - G_{m4}V_{i3}) + G_{m1}G_{m2}G_{m4}V_{i2}]$$

$$V_{o2} = [1/D(s)] \cdot [(sC_1G_{m2}G_{m5} + G_{m1}G_{m2}G_{m3})V_{i2}G_{m1}G_{m2}(G_{m4}V_{i3} - G_{m5}V_{i1})]$$

$$V_{o3} = [1/D(s)] \cdot [s^2C_1C_2G_{m4}V_{i3} + s(C_2G_{m1}G_{m3}V_{i1} - C_1G_{m2}G_{m4}V_{i2}) + G_{m1}G_{m2}G_{m4}V_{i1}]$$

$$D(s) = C_1C_2G_{m5}\left(s^2 + s\frac{1}{C_1}\frac{G_{m1}G_{m3}}{G_{m5}} + \frac{G_{m1}G_{m2}G_{m4}}{C_1C_2G_{m5}}\right)$$

If $V_{i1} = V_{i2} = 0$, then

$$\frac{V_{o1}}{V_{i3}} = H_{BP}(s) = -\frac{sC_2G_{m1}G_{m4}}{D(s)}$$

$$\frac{V_{o2}}{V_{i3}} = H_{LP}(s) = \frac{G_{m1}G_{m2}G_{m4}}{D(s)}$$

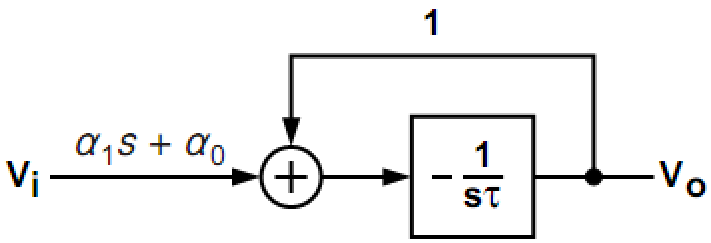
$$\frac{V_{o3}}{V_{i3}} = H_{HP}(s) = \frac{s^2C_1C_2G_{m4}}{D(s)}$$

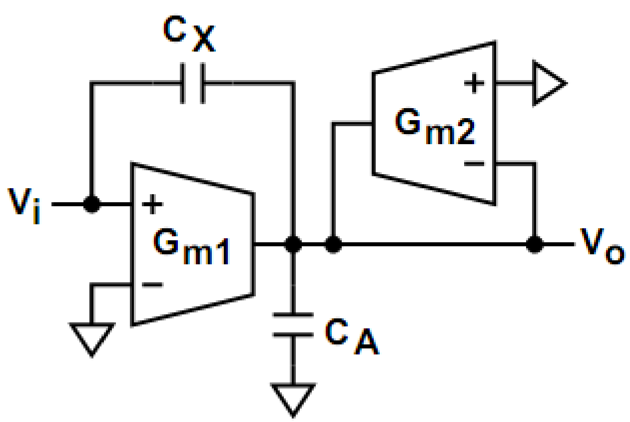
If $V_{i1} = V_{i2} = V_{i3} = V_i$, then

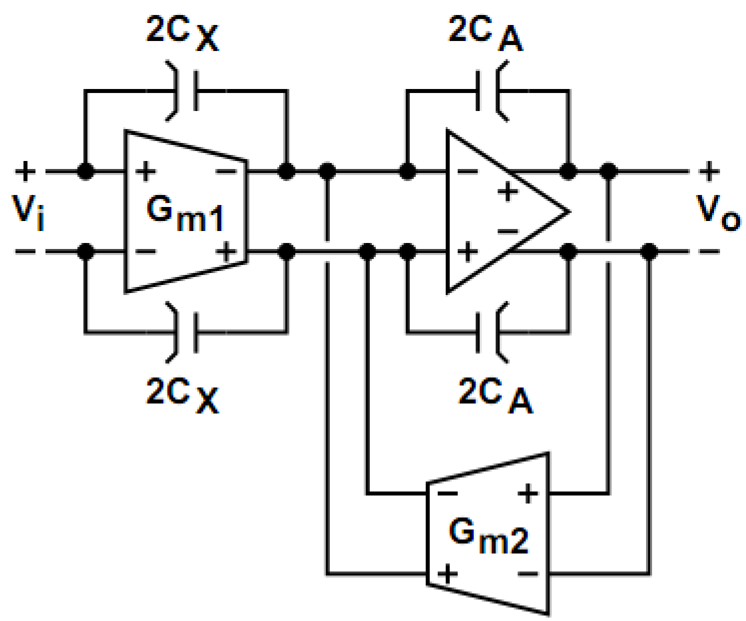
$$\frac{V_{o3}}{V_i} = \frac{s^2C_1C_2G_{m4} + s(C_2G_{m1}G_{m3} - C_1G_{m2}G_{m4}) + G_{m1}G_{m2}G_{m4}}{D(s)}$$

- If $C_2G_{m1}G_{m3} = C_1G_{m2}G_{m4}$, it is a band-reject biquad.
- If $C_1G_{m2}G_{m4} = 2C_2G_{m1}G_{m3}$ and $G_{m4} = G_{m5}$, it is an allpass biquad.
- There is one parasitic pole in the biquad.

Gm-C 1st-order filter using Miller integrators







Gm-C 1st-order filter using Miller integrators

Without the Miller Integrator

$$\frac{V_o}{V_i} = \frac{\alpha_1 s + \alpha_0}{s + \omega_o} = \frac{s\left(\frac{C_X}{C_A+C_X}\right) + \left(\frac{G_{m1}}{C_A+C_X}\right)}{s + \left(\frac{G_{m2}}{C_A+C_X}\right)}$$

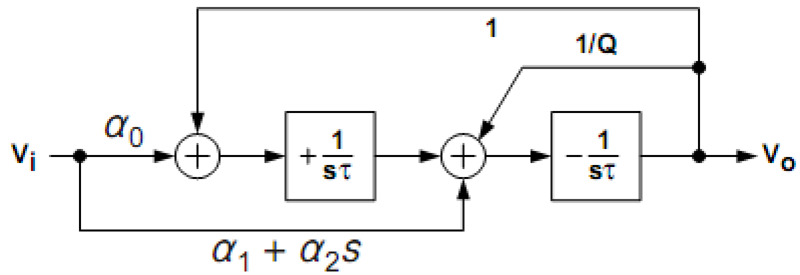
$$G_{m1} = \alpha_0(C_A + C_X) \qquad G_{m2} = \omega_o(C_A + C_X) \qquad C_X = C_A\frac{\alpha_1}{1-\alpha_1} \quad \text{where} \quad 0 \le \alpha_1 < 1$$

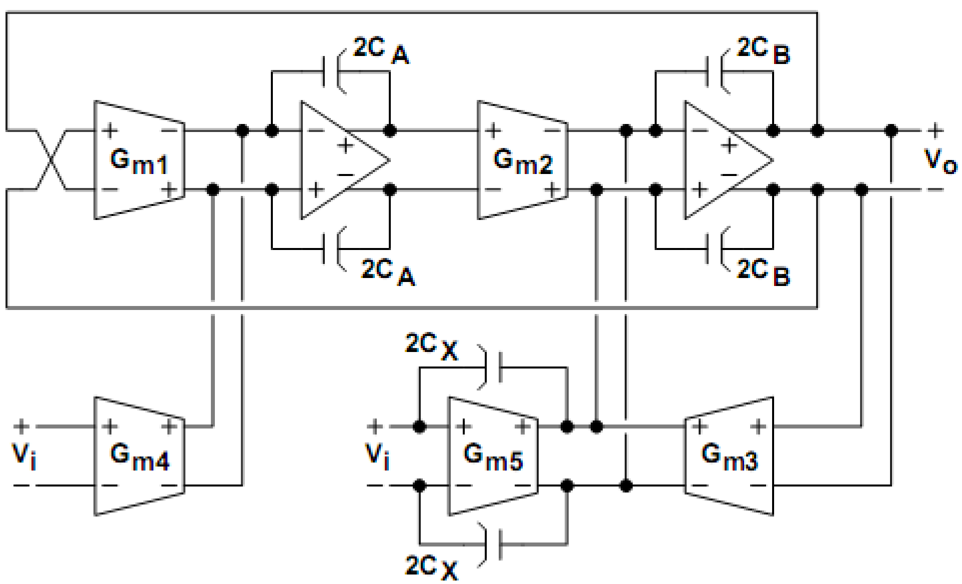
With the Miller Integrator

$$\frac{V_o}{V_i} = \frac{\alpha_1 s + \alpha_0}{s + \omega_o} = \frac{s\left(\frac{C_X}{C_A}\right) + \left(\frac{G_{m1}}{C_A}\right)}{s + \left(\frac{G_{m2}}{C_A}\right)}$$

- The use of feed-in capacitors can simplify design, but requires inputs of low source impedance.

Gm-C 2nd -order filter using Miller integrators





Gm-C 2nd -order filter using Miller integrators

The transfer function is

$$\frac{V_o}{V_i} = \frac{\alpha_2 s^2 + \alpha_1 s + \alpha_o}{s^2 + \left(\frac{\omega_p}{Q}\right) + \omega_p^2} = \frac{s^2\left(\frac{C_X}{C_B}\right) + s\left(\frac{G_{m5}}{C_B}\right) + \left(\frac{G_{m2}G_{m4}}{C_A C_B}\right)}{s^2 + s\left(\frac{G_{m3}}{C_B}\right) + \left(\frac{G_{m1}G_{m2}}{C_A C_B}\right)}$$

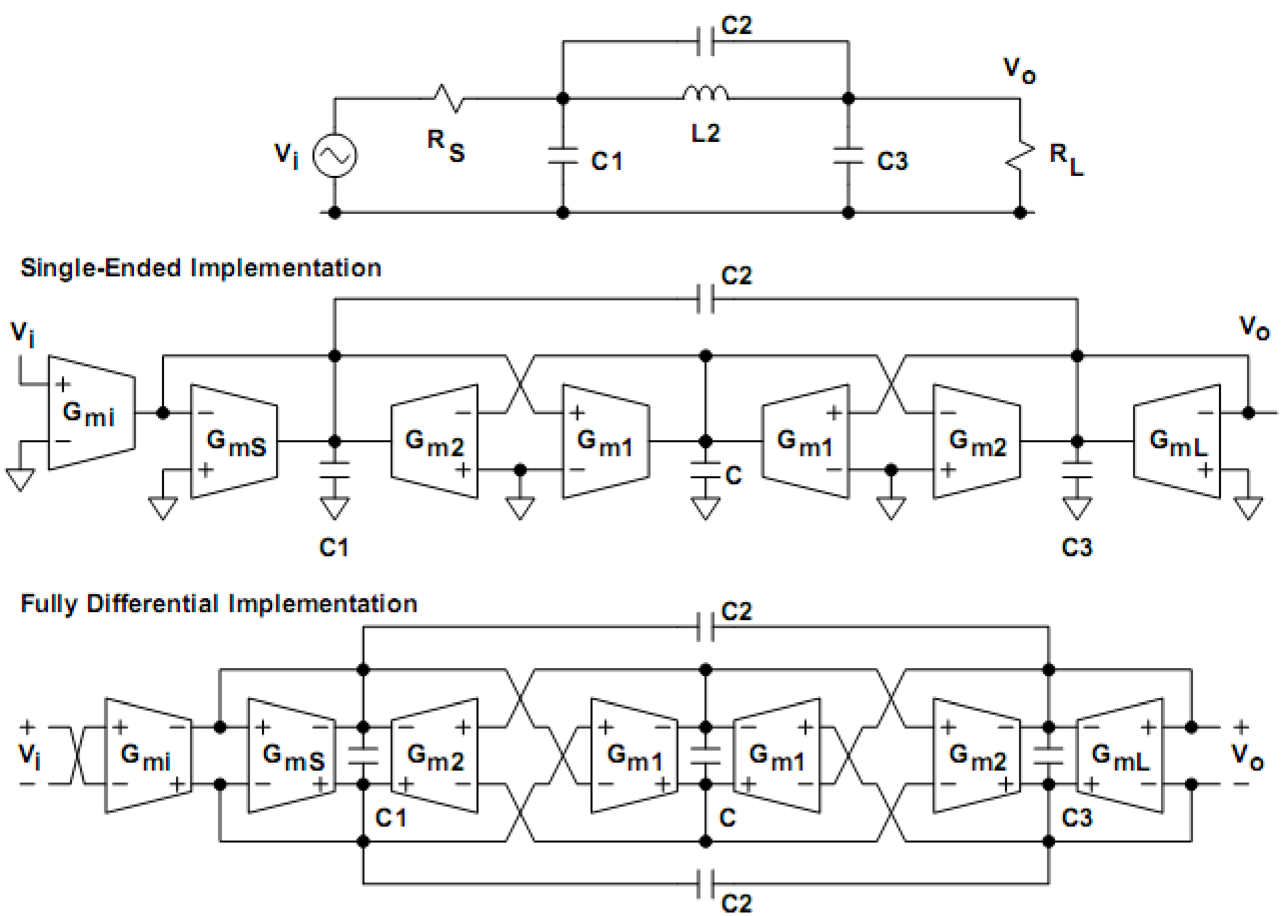
Thus

$$C_X = \alpha_2 C_B$$

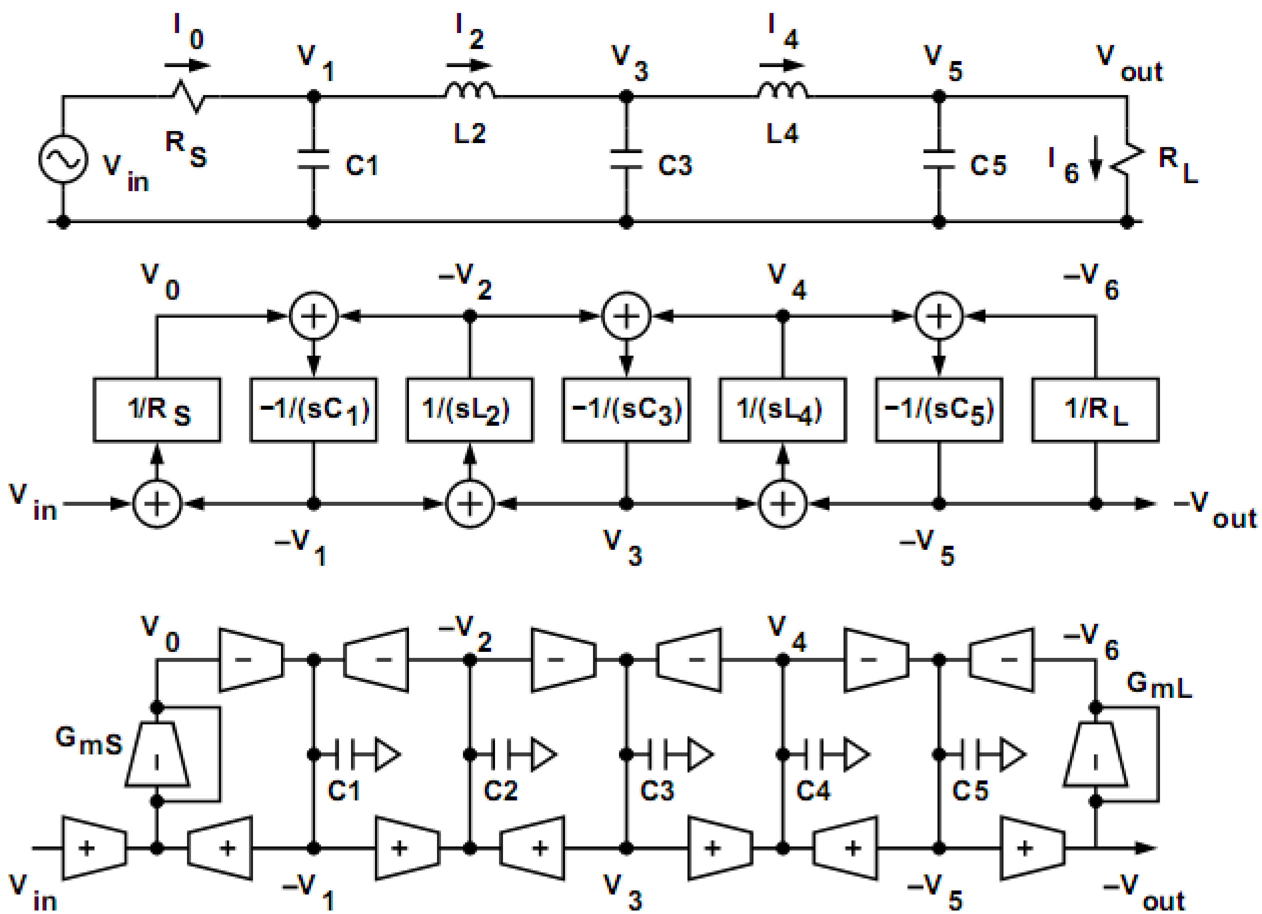
and

$$G_{m1} = \omega_p C_A \qquad G_{m2} = \omega_p C_B \qquad G_{m3} = \frac{\omega_p C_B}{Q} \qquad G_{m4} = \frac{\alpha_0 C_A}{\omega_p} \qquad G_{m5} = \alpha_1 C_B$$

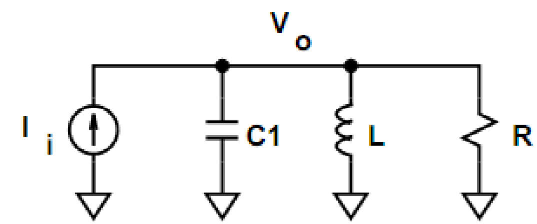
Ladder filter using Gm-C gyrators

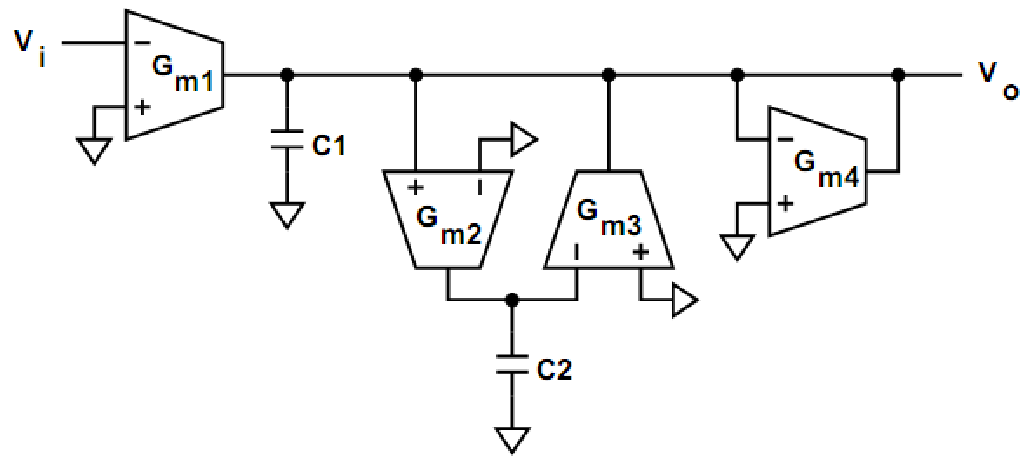


Ladder filter using Signal-Flow Graph



Gm-C resonator





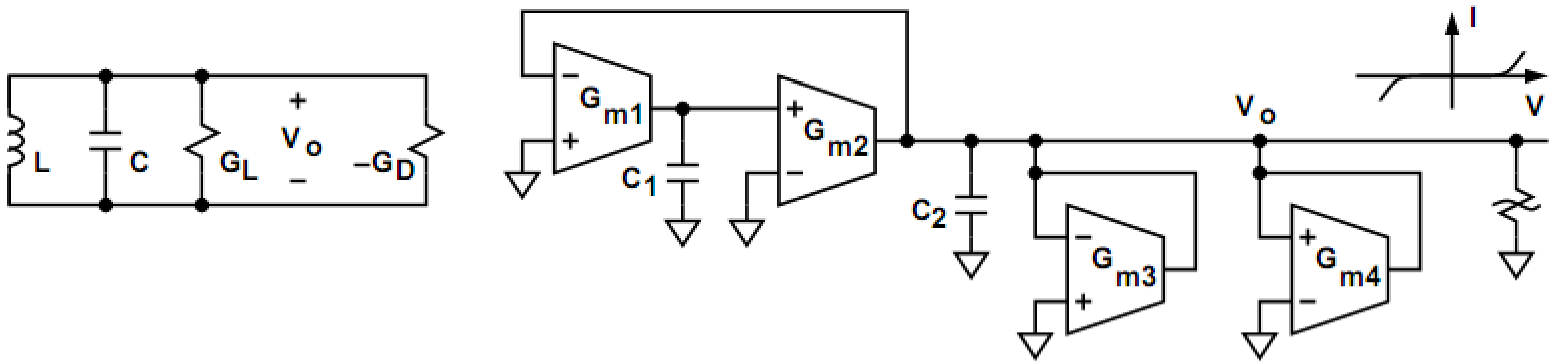
- The resonant frequency and the quality factor are

$$\omega_o = \sqrt{\frac{1}{LC_1}} = \sqrt{\frac{G_{m2}G_{m3}}{C_1C_2}} \qquad Q = \omega_o RC_1 = \sqrt{\frac{C_1}{C_2}} \times \sqrt{\frac{G_{m2}G_{m3}}{G_{m4}^2}}$$

The voltage gain at the resonant frequency is

$$A_{vo} = \frac{v_o}{v_i} = G_{m1}R = \frac{G_{m1}}{G_{m4}}$$

Gm-C oscillator



- The combination of G_{m1}, G_{m2} and C_1 simulates an inductor.
- The oscillation frequency is $\omega_o = \sqrt{G_{m1}G_{m2}/(C_1C_2)}$.
- The oscillation condition is $G_{m4} = G_{m3}$. In many cases, G_{m3} and G_{m4} are not required.
- The nonlinear resistor is used to control the output amplitude.