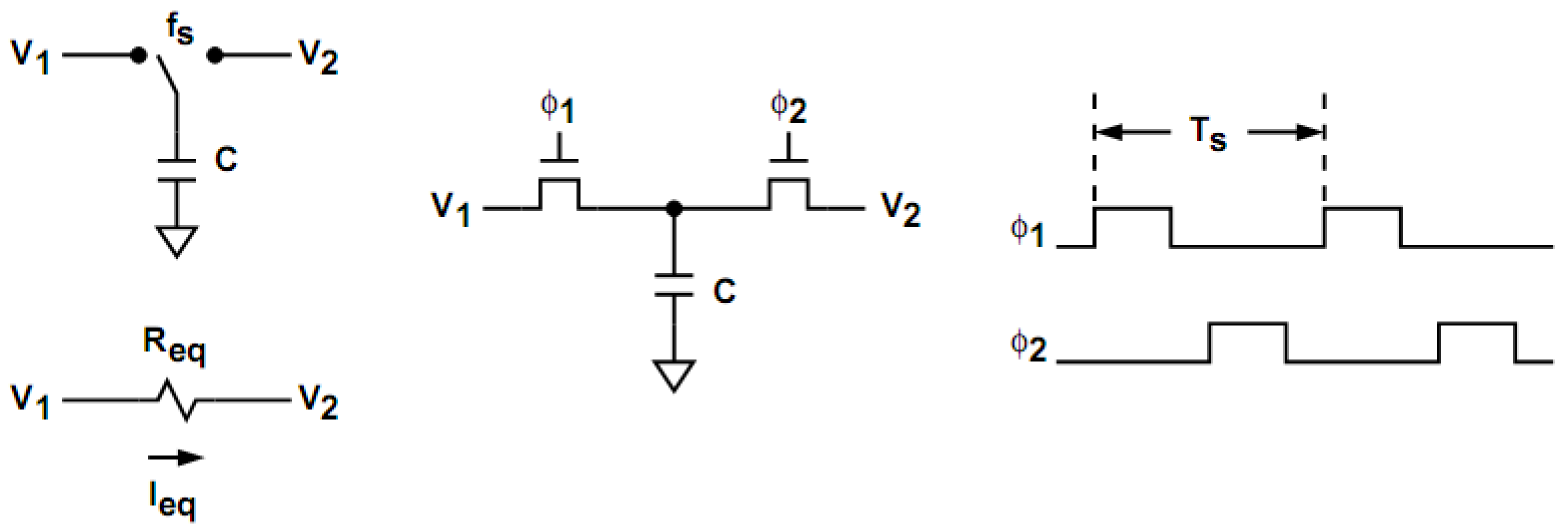
Switched-capacitor (SC) filters

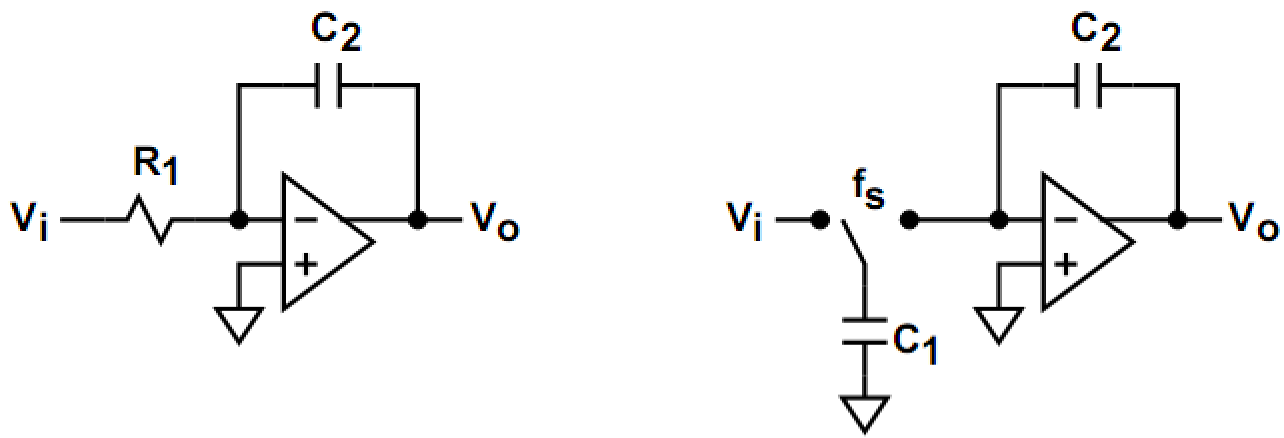
- SC resistor



$$I_{eq} = \overline{\left(\frac{\Delta Q}{\Delta t}\right)} = \frac{C \cdot V_1 - C \cdot V_2}{T_s} = C \cdot (V_1 - V_2) \cdot f_s \qquad T_s = \frac{1}{f_s}$$

$$G_{eq} = \frac{1}{R_{eq}} = \frac{I_{eq}}{V_1 - V_2} = C \cdot f_s$$

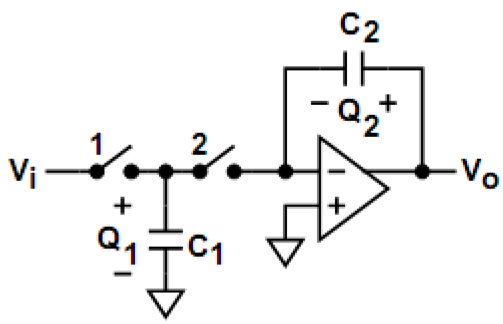
SC integrator

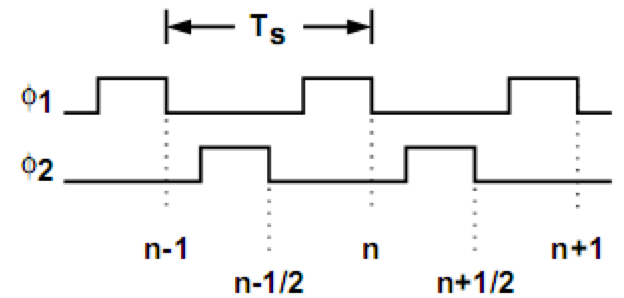


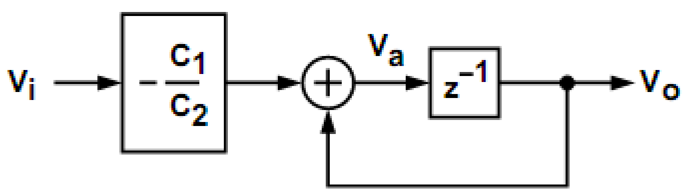
$$\frac{V_o}{V_i} = -\frac{1}{sR_1C_2} = -\frac{1}{s}\cdot\frac{G_{eq1}}{C_2} = -\frac{1}{s}\cdot\left(f_s\cdot\frac{C_1}{C_2}\right)$$

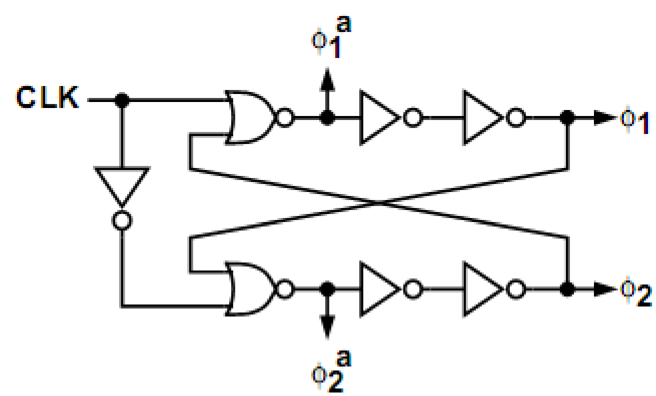
- Consist of analog switches, capacitors and opamps.
- Discrete-time (or sampled-data) analog filters.
- Time constant is determined by capacitance ratio and switching frequency.

SC integrator analysis









$$\frac{V_o(z)}{V_i(z)} = -\frac{C_1}{C_2} \times \frac{z^{-1}}{1 - z^{-1}}$$

SC integrator analysis

At cycle n, i.e., $t = nT_s$, we have $Q_1(n) = C_1V_i(n)$ and $Q_2(n) = C_2V_o(n)$

At cycle $n + 1/2$, i.e., $t = (n + 1/2)T_s$,

$$Q_1(n + 1/2) = 0 \qquad Q_2(n + 1/2) = Q_2(n) - Q_1(n) = C_2V_o(n) - C_1V_i(n)$$

At cycle $n + 1$, i.e., $t = (n + 1)T_s$,

$$Q_1(n + 1) = C_1V_i(n + 1) \qquad Q_2(n + 1) = C_2V_o(n + 1) = Q_2(n + 1/2) = C_2V_o(n) - C_1V_i(n)$$

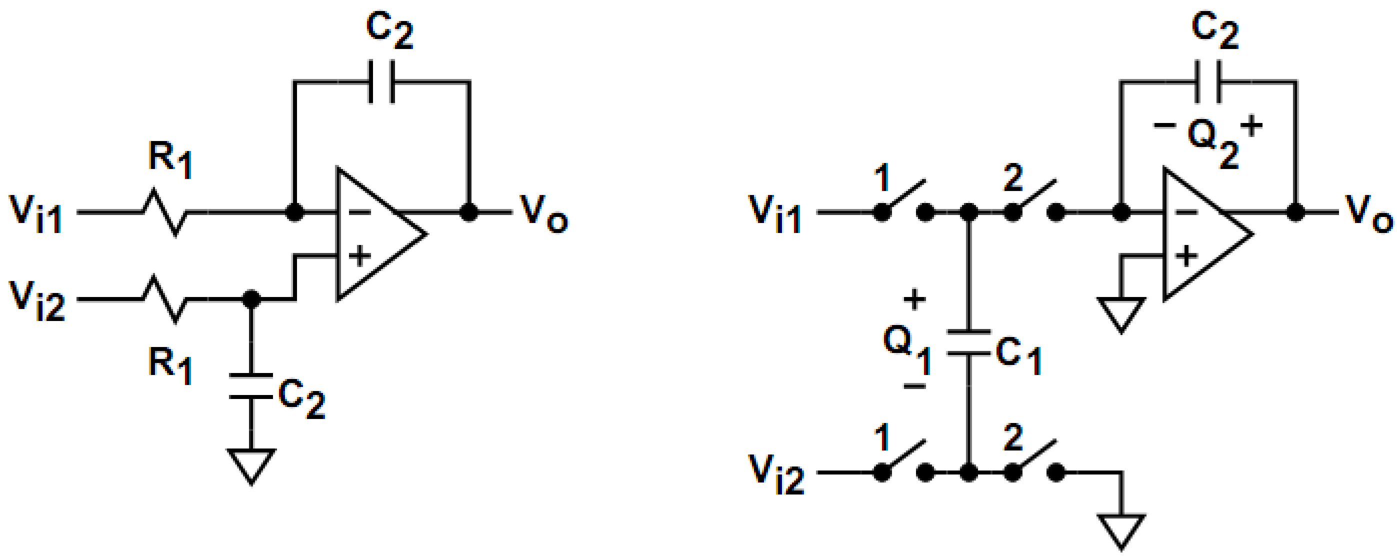
Thus, the time-domain difference equation is

$$C_2V_o(n + 1) = C_2V_o(n) - C_1V_i(n)$$

In the z-domain

$$zC_2V_o(z) = C_2V_o(z) - C_1V_i(z) \qquad \Rightarrow \qquad \frac{V_o(z)}{V_i(z)} = -\frac{C_1}{C_2} \times \frac{1}{z - 1} = -\frac{C_1}{C_2} \times \frac{z^{-1}}{1 - z^{-1}}$$

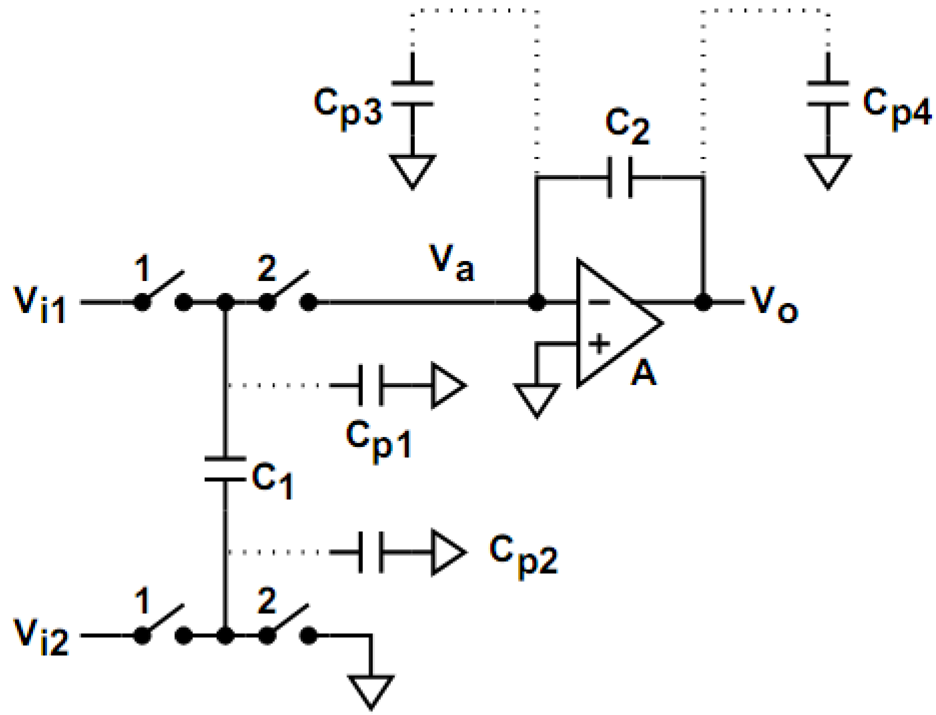
SC differential integrator



RC Integrator → $$V_o(s) = -\frac{1}{sR_1C_2}(V_{i1} - V_{i2})$$

SC Integrator → $$V_o(z) = -\frac{C_1}{C_2} \times \frac{z^{-1}}{1 - z^{-1}} \times [V_{i1}(z) - V_{i2}(z)]$$

Effects of parasitic Cs



$$V_o(z) = \left[-\frac{C_1}{C_2}[V_{i1}(z) - V_{i2}(z)] - \frac{C_{p1}}{C_2}V_{i1}(z) \right] \times \frac{z^{-1}}{1 - z^{-1}}$$

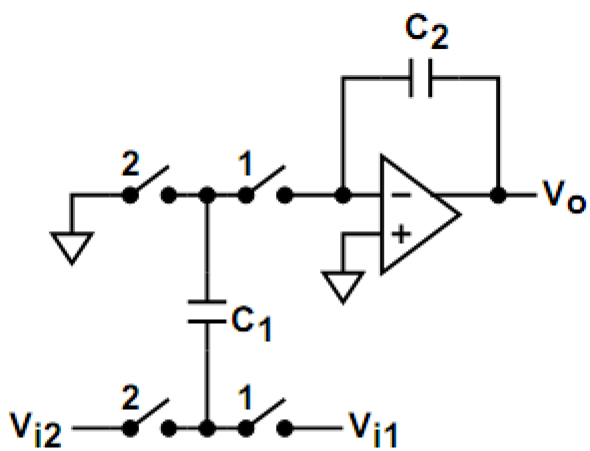
Effects of parasitic Cs

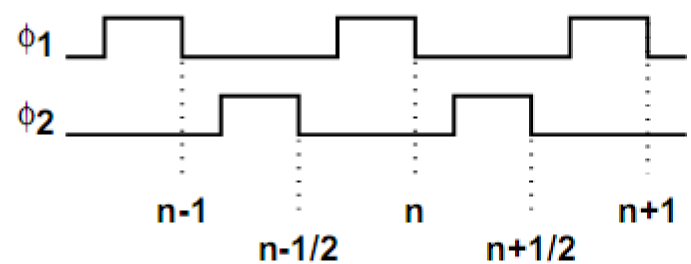
- Among the parasitic capacitors, only C_{p1} contribute charge to C_2 if $A = \infty$.
- Consider a finite value of A, then $V_o = -A \cdot V_a$, and

$$C_1[V_{i1}(n) - V_{i2}(n)] + C_2[V_a(n) - V_o(n)] + C_{p1}V_{i1} + C_{p3}V_a(n)$$
$$= \left(C_1 + C_{p1} + C_{p3}\right) V_a(n+1) + C_2[V_a(n+1) - V_o(n+1)]$$
$$\Rightarrow \qquad V_o(z) = \frac{\left[-\frac{C_1}{C_2}[V_{i1}(z) - V_{i2}(z)] - \frac{C_{p1}}{C_2}V_{i1}(z)\right] \times z^{-1}}{1 + \frac{1}{A}\left(1 + \frac{C_1}{C_2} + \frac{C_{p1}}{C_2} + \frac{C_{p3}}{C_2}\right) - z^{-1}\left[1 + \frac{1}{A}\left(1 + \frac{C_{p3}}{C_2}\right)\right]}$$

- Must keep $C_{p1} \ll C_1$ and $C_{p1,p3} \ll C_2$.
  - Connect the top plates of the capacitors to the opamp's input.
  - Let the bottom plates of the capacitors always be driven.

Parasitic-insensitive SC integrator

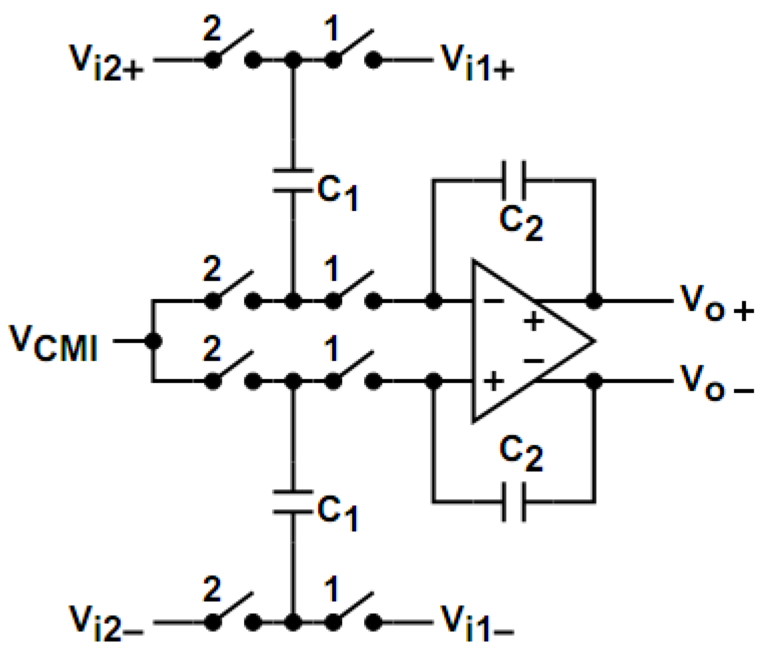




$$V_o(z) = \frac{\frac{C_1}{C_2}\left[-V_{i1} + z^{-1}V_{i2}\right]}{1 + \frac{1}{A}\left(1 + \frac{C_1}{C_2} + \frac{C_{p1}}{C_2} + \frac{C_{p3}}{C_2}\right) - z^{-1}\left[1 + \frac{1}{A}\left(1 + \frac{C_{p3}}{C_2}\right)\right]}$$

- Insensitive to parasitics if $A \to \infty$.
- The two inputs have different delays.

Fully-differential SC integrator

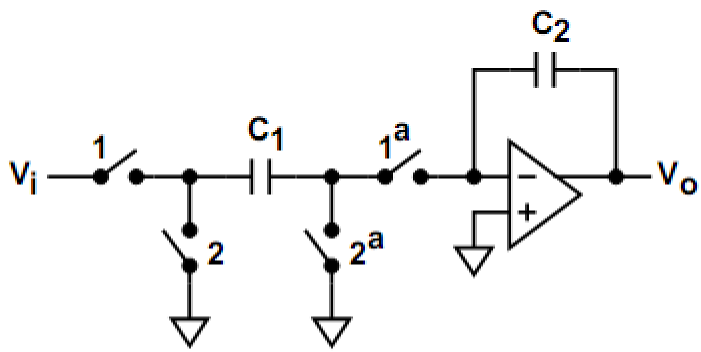


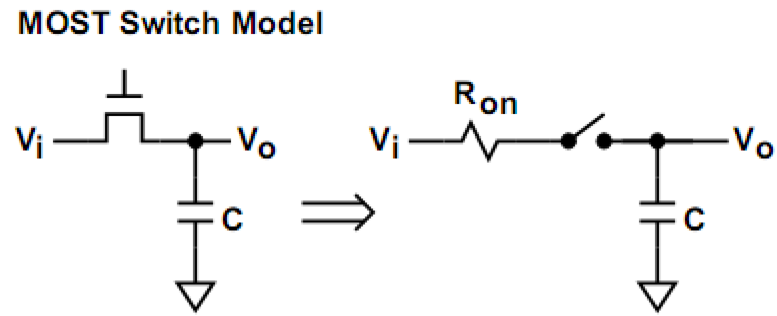
$$V_{i1} = V_{i1+} - V_{i1-}$$
$$V_{i2} = V_{i2+} - V_{i2-}$$
$$V_o = V_{o+} - V_{o-}$$

$$V_o(z) = \frac{C_1}{C_2} \times \left[-\frac{1}{1 - z^{-1}} \cdot V_{i1} + \frac{z^{-1}}{1 - z^{-1}} \cdot V_{i2} \right]$$

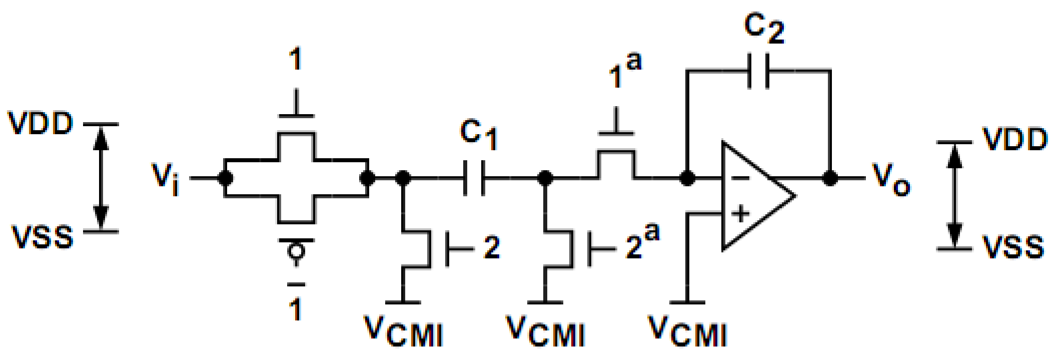
- V_{CMI} and V_{CMO} can be different.

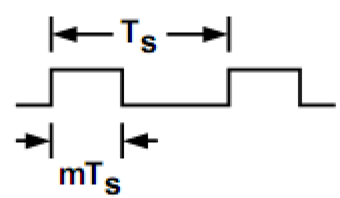
MOSFET switch





$$R_{on} = \frac{1}{\mu C_{ox}(W/L)V_{ov}}$$





For good settling, want

$$mT_s > 5R_{on}C = \frac{5C}{\mu C_{ox}(W/L)V_{ov}}$$

MOSFET switch

- When turning off the switch, the switching error is

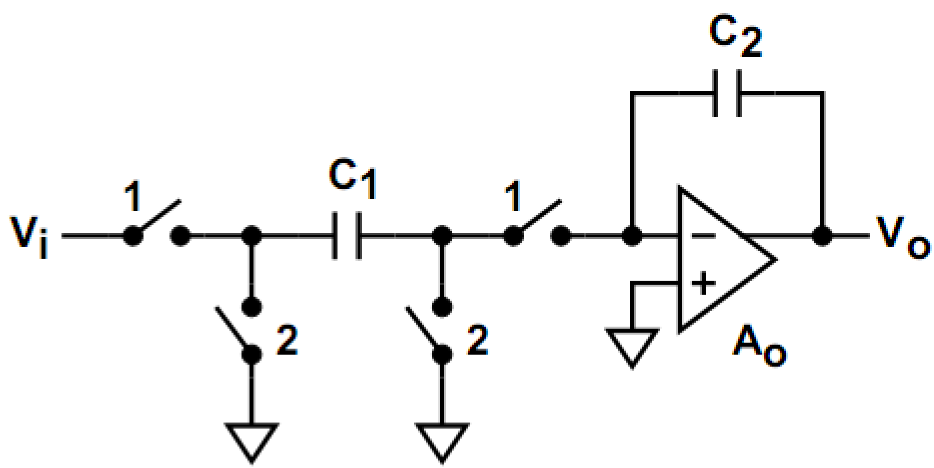
$$\Delta V = \frac{\alpha Q_{CH}}{C} = \frac{\alpha W L C_{ox} V_{ov}}{C}$$

The maximum clock rate is

$$f_s < \frac{m}{\alpha} \cdot \frac{\mu \Delta V}{5L^2}$$

- Realize switches connected to V_{SS} or near V_{SS} with nMOSTs.
- Realize switches connected to V_{DD} or near V_{DD} with pMOSTs.
- Turn off the switches near the virtual ground node of the opamps first.
- The thermal noise is proportional to kT/C.
- There are also noises from the power supplies.

Effects of opamp finite DC gain



If $A_o = \infty$, then

$$V_o(n) = -kV_i(n) + V_o(n-1)$$

$$H(z) = -\frac{k}{1 - z^{-1}}$$

$$k = \frac{C_1}{C_2}$$

If $A_o = 1/\mu$ is finite, then

$$V_o(n) = -k\alpha V_i(n) + \beta V_o(n-1) \qquad H(z) = -\frac{k\alpha}{1 - \beta z^{-1}}$$

$$\alpha = \frac{1}{1 + (1+k)\mu} \approx 1 - (1+k)\mu = 1 + \Delta\alpha \qquad \Delta\alpha = -(1+k)\mu \ll 1$$

$$\beta = \frac{1+\mu}{1 + (1+k)\mu} \approx 1 - k\mu = 1 + \Delta\beta \qquad \Delta\beta = -k\mu \ll 1$$

Effects of opamp finite DC gain

The transfer function $H(z)$ in s-domain is

$$H(e^{j\omega T_s}) \approx -\frac{k}{1-z^{-1}}\bigg|_{z=e^{j\omega T_s}} \times [1+m(\omega)]e^{j\theta(\omega)}$$

$$m(\omega) \approx \Delta\alpha - \frac{\Delta\beta}{2} \approx -\left(1+\frac{1}{2}\frac{C_1}{C_2}\right)\cdot\frac{1}{A_o}$$

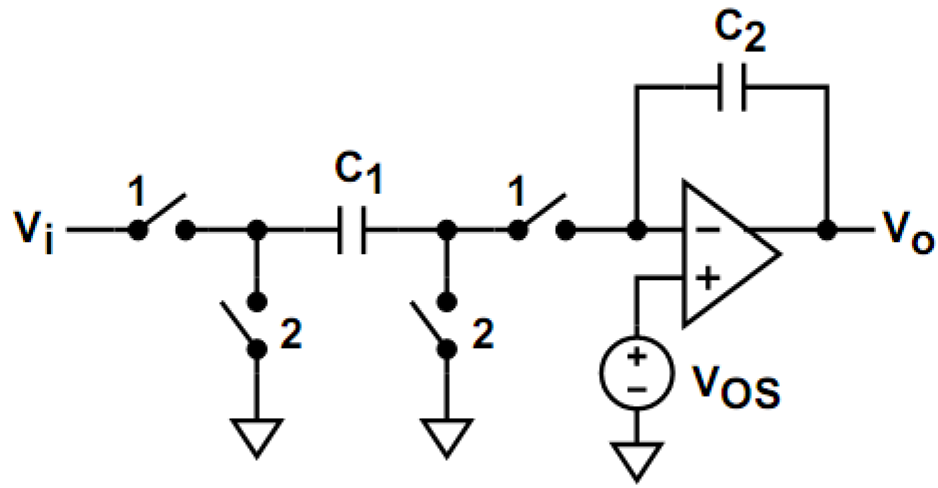
$$\theta(\omega) \approx -\frac{\Delta\beta}{2}\cdot\frac{1}{\tan(\omega T_s/2)} \approx \frac{1}{2}\cdot\frac{C_1}{C_2}\cdot\frac{1}{A_o}\cdot\frac{1}{\tan(\omega T_s/2)} \approx \frac{C_1}{C_2}\cdot\frac{1}{A_o}\cdot\frac{1}{\omega T_s}$$

- At the unit-gain frequency ω_i, where $\left|H(e^{j\omega_i T_s})\right| = 1$, we have

$$-m(\omega_i) \approx \theta(\omega_i) \approx 1/A_o \qquad \text{if} \qquad \omega_i T_s/2 \ll 1$$

- In most applications, the magnitude error $m(\omega)$ has negligible effect, but the phase error $\theta(\omega)$ can be detrimental in narrowband (high-Q) filters.

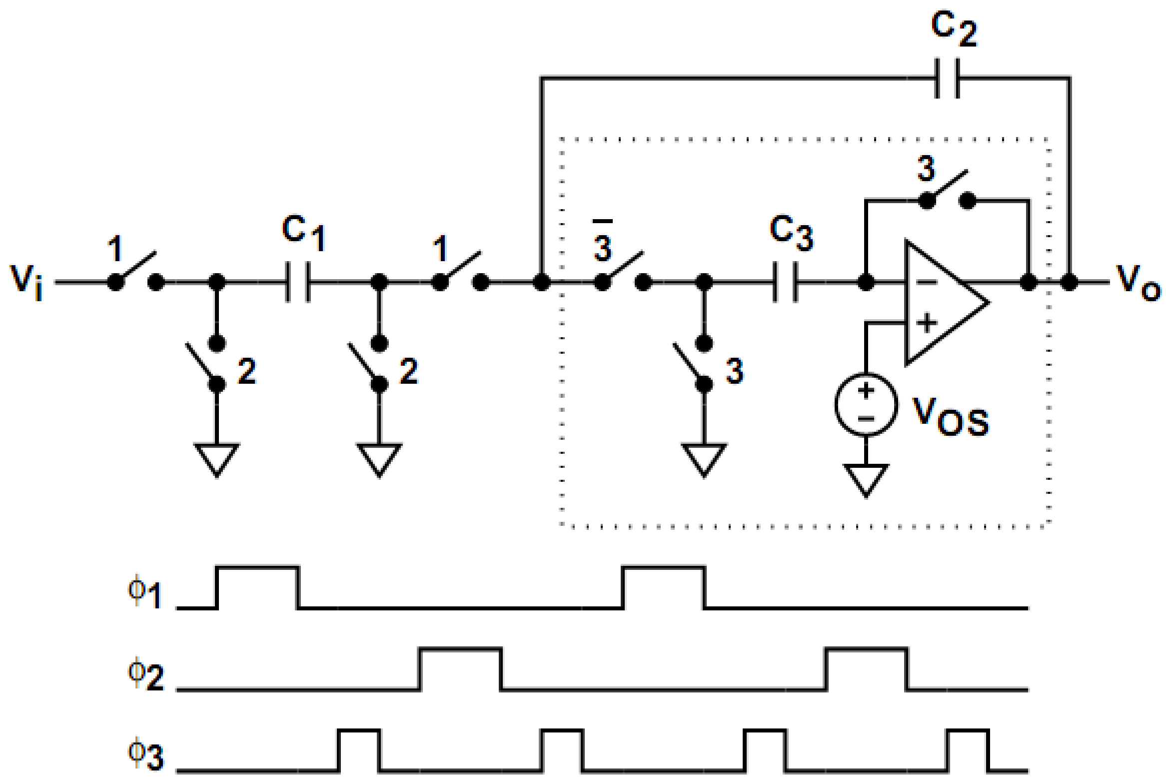
Effects of opamp DC offset



$$V_o(z) = -\frac{C_1}{C_2}\frac{1}{1-z^{-1}} \cdot V_i(z) + \frac{C_1}{C_2}\frac{1}{1-z^{-1}} \cdot V_{OS} + V_{OS}$$

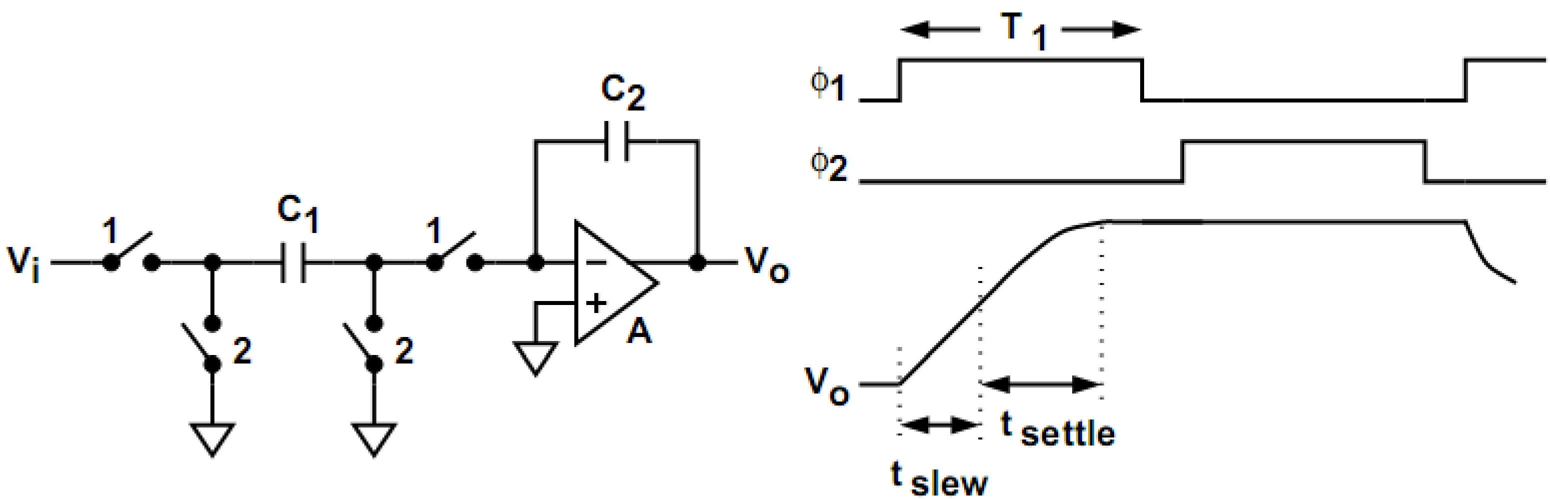
- The V_{OS} to V_o transfer function is also an integration.
- When the entire filter is considered, the V_{OS} may cause finite dc level shift in this and other integrators.

Auto-zeroing scheme



- During the ϕ_3 auto-zeroing mode, opamp's offset voltage is stored in C_3.

Effect of opamp finite settling time

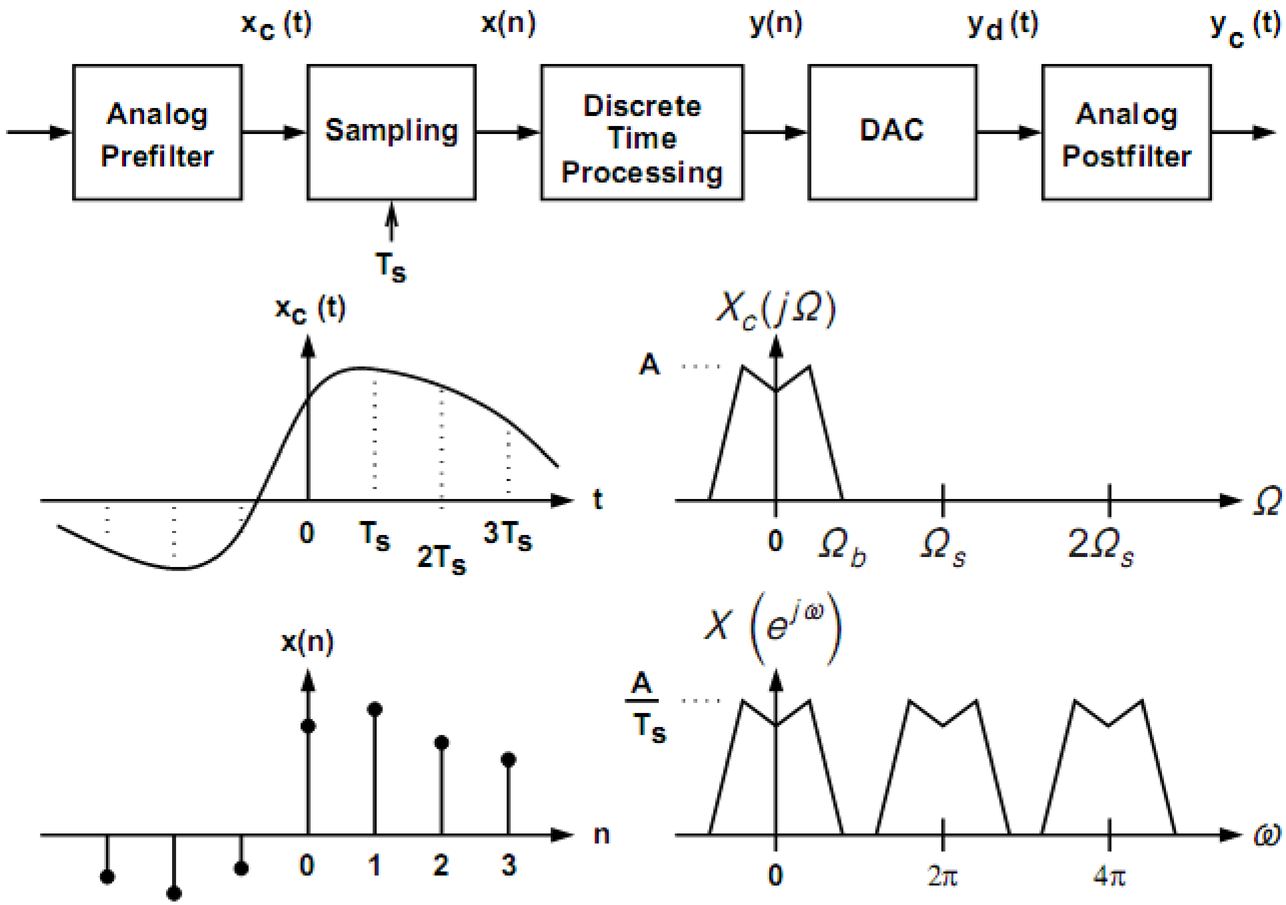


- Let $t_{slew} = 0$, $A(s) = \omega_u/s$, $T_1 = T_s/2$, ω_i is the unit-gain frequency of the integrator, and $\omega_i T_s \ll 1$. At $\omega = \omega_i$, the magnitude error and phase error of the integrator are

$$m(\omega_i) \approx \theta(\omega_i) \approx -\omega_i T_s e^{-\omega_u T_s/2}$$

- Want $\omega_u \geq 5 \cdot \omega_s$. However, to avoid unnecessary noise aliasing, ω_u should not be too much larger than necessary.

Discrete-time signal processing



Continuous-time signal

The Laplace transform and the continuous-time Fourier transform (CTFT) are

$$X_c(s) = \int_{-\infty}^{\infty} x_c(t)e^{-st}dt \qquad X_c(j\Omega) = \int_{-\infty}^{\infty} x_c(t)e^{-j\Omega t}dt$$

If the region of convergence of $X_c(s)$ includes the imaginary axis, then

$$X_c(j\Omega) = X_c(s)|_{s=j\Omega}$$

Sampling Theorem: To avoid aliasing, want

$$\Omega_s > 2\Omega_b \qquad \Omega_s = 2\pi f_s = \frac{2\pi}{T_s}$$

- Ω_b is the bandwidth of $x_c(t)$, Ω_s is the sampling frequency, and $2\Omega_b$ is called the *Nyquist rate.*

Discrete-time signal

In discrete-time domain, the z transform is

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

The discrete-time Fourier transform (DTFT) is

$$X\left(e^{j\omega}\right) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$

If the region of convergence of $X(z)$ includes the unit circle, then

$$X\left(e^{j\omega}\right) = X(z)\big|_{z=e^{j\omega}}$$

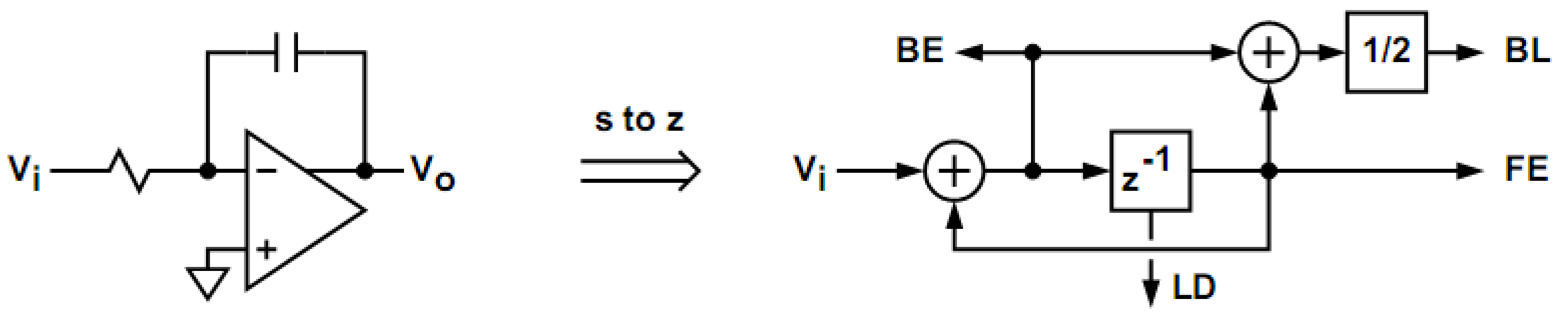
s-to-z transformation

Want to approximate $H_c(s)$ with $H(z)$.

$$z = e^{sT_s} \qquad s = \frac{1}{T_s} \cdot \ln z \qquad \Rightarrow \qquad H(z) = H_c(s)|_{s=(1/T_s)\ln z} \approx H_c(s)|_{s=T(z)}$$

Transformation error of an Integrator can be written as

$$H_c(s) = \frac{1}{s} = \frac{1}{j\Omega} \qquad \Rightarrow \qquad H(z)|_{e^{j\Omega T_s}} = \frac{1}{j\Omega} \cdot [1 - \epsilon(\Omega)] \cdot e^{j\phi(\Omega)}$$



s-to-z transformation

Backward Euler (BE) Transformation

$$s = \frac{1}{T_s} \cdot (1 - z^{-1}) \quad \Rightarrow \quad \frac{1}{s} = T_s \cdot \frac{1}{1 - z^{-1}} \qquad \epsilon = 1 - \frac{\Omega T_s/2}{\sin(\Omega T_s/2)} \qquad \phi = +\frac{\Omega T_s}{2}$$

Forward Euler (FE) Transformation

$$s = \frac{1}{T_s} \cdot \frac{1 - z^{-1}}{z^{-1}} \quad \Rightarrow \quad \frac{1}{s} = T_s \cdot \frac{z^{-1}}{1 - z^{-1}} \qquad \epsilon = 1 - \frac{\Omega T_s/2}{\sin(\Omega T_s/2)} \qquad \phi = -\frac{\Omega T_s}{2}$$

Lossless Discrete (LD) Transformation

$$s = \frac{1}{T_s} \cdot \frac{1 - z^{-1}}{z^{-1/2}} \quad \Rightarrow \quad \frac{1}{s} = T_s \cdot \frac{z^{-1/2}}{1 - z^{-1}} \qquad \epsilon = 1 - \frac{\Omega T_s/2}{\sin(\Omega T_s/2)} \qquad \phi = 0$$

s-to-z transformation

The transformation is

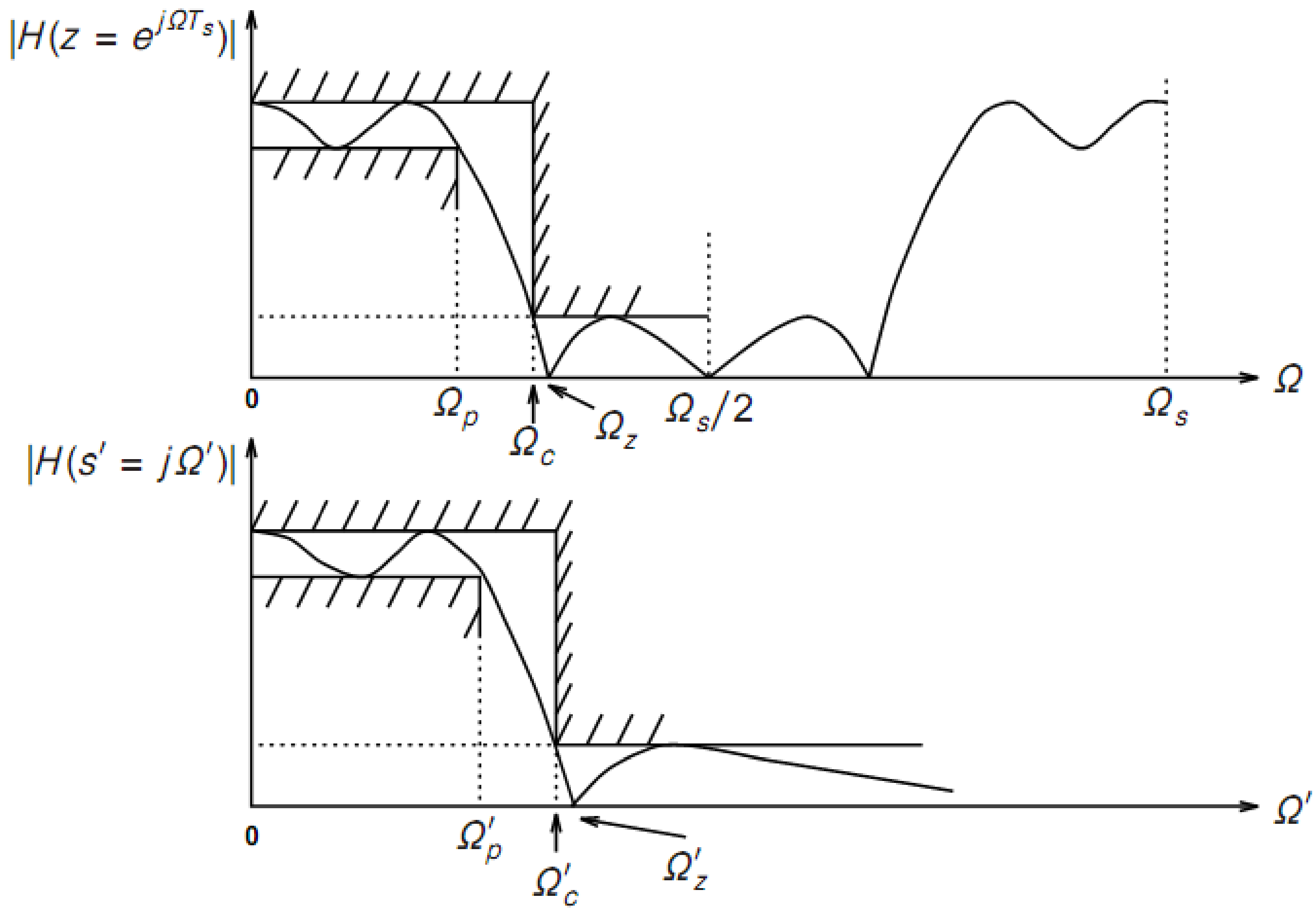
$$s = \frac{2}{T_s} \cdot \frac{1 - z^{-1}}{1 + z^{-1}} \qquad \Rightarrow \qquad \frac{1}{s} = \frac{T_s}{2} \cdot \frac{1 + z^{-1}}{1 - z^{-1}} \qquad \epsilon = 1 - \frac{\Omega T_s/2}{\tan(\Omega T_s/2)} \qquad \phi = 0$$

let $z = e^{j\omega}$, then

$$s = \frac{2}{T_s} \cdot \frac{e^{j\omega} - 1}{e^{j\omega} + 1} = \frac{2}{T_s} \cdot j \tan\left(\frac{\omega}{2}\right) = j\Omega \qquad \Omega = \frac{2}{T_s} \tan\left(\frac{\omega}{2}\right)$$

- The unit circle in the z-plane is mapped to the $j\Omega$ axis in the s-plane.

Bilinear transformation



Bilinear transformation

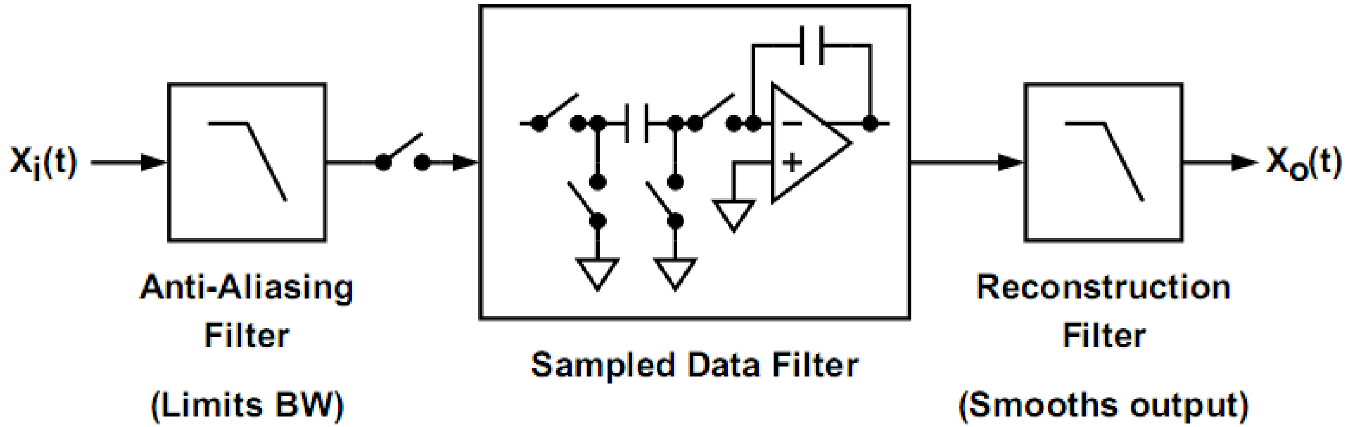
- Prewarp the filter specifications from Ω to Ω'.

$$\Omega'_p = \frac{2}{T_s}\tan\frac{\Omega_p T_s}{2} \qquad \Omega'_c = \frac{2}{T_s}\tan\frac{\Omega_c T_s}{2} \qquad \Omega'_z = \frac{2}{T_s}\tan\frac{\Omega_z T_s}{2}$$

- Find $H_c(s')$.
- The $H(z)$ is obtained by

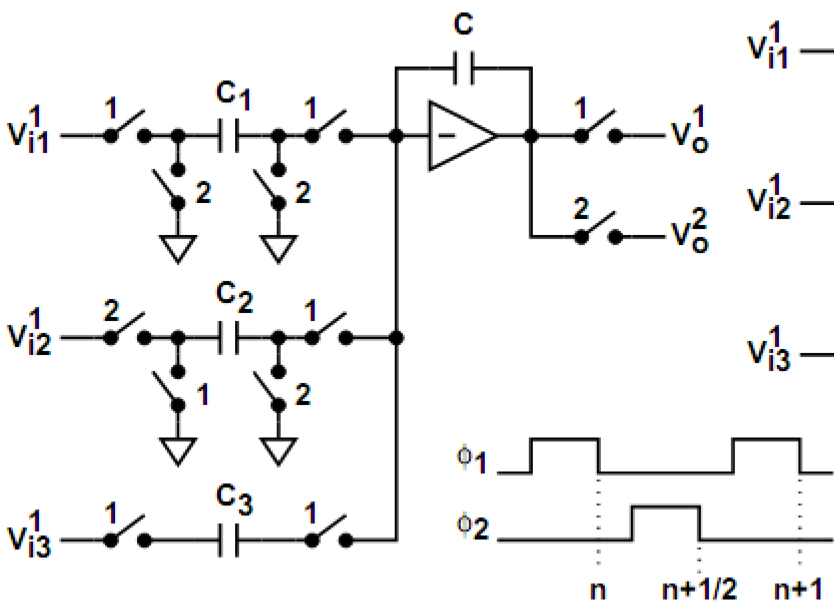
$$H(z) = H_c\left(s' = \frac{2}{T_s}\cdot\frac{1-z^{-1}}{1+z^{-1}}\right)$$

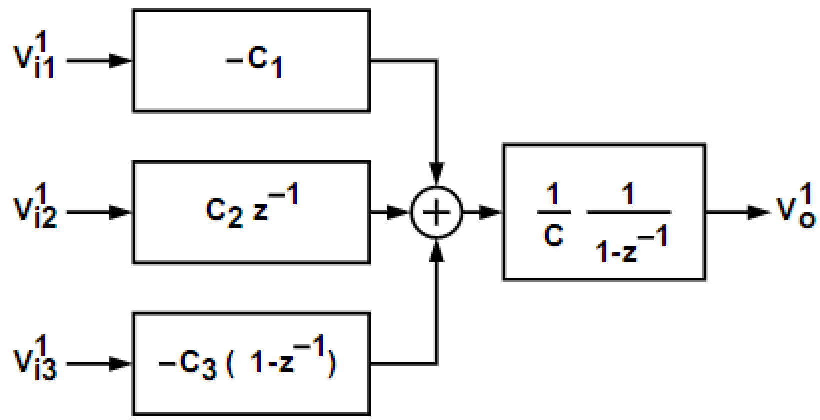
SC filter system



- Discrete-time (or sampled-data) analog filters.
- Filters consist of analog switches, capacitors and opamps.
- Filter response is determined by ratios of capacitance.
- Switched-C “resistor” cannot be the only feedback around an opamp. Since the path is not continuous, it won’t stabilize the opamp.
- No floating node. Otherwise charge can accumulate.
- Capacitor bottom plate must always be driven from a low impedance (voltage sources or ground).
- Connect non-inverting opamp input to a dc bias. Otherwise response is sensitive to parasitic capacitances.

Active SC integrator

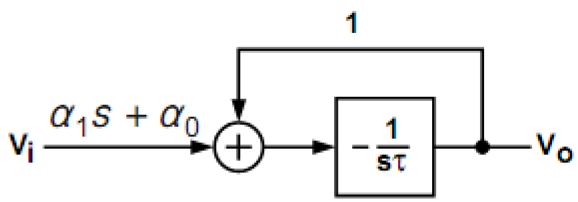


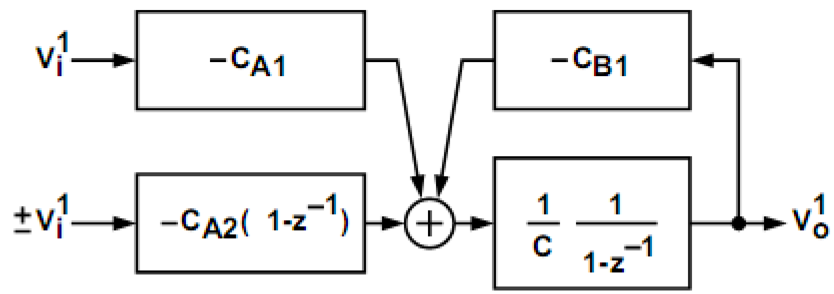


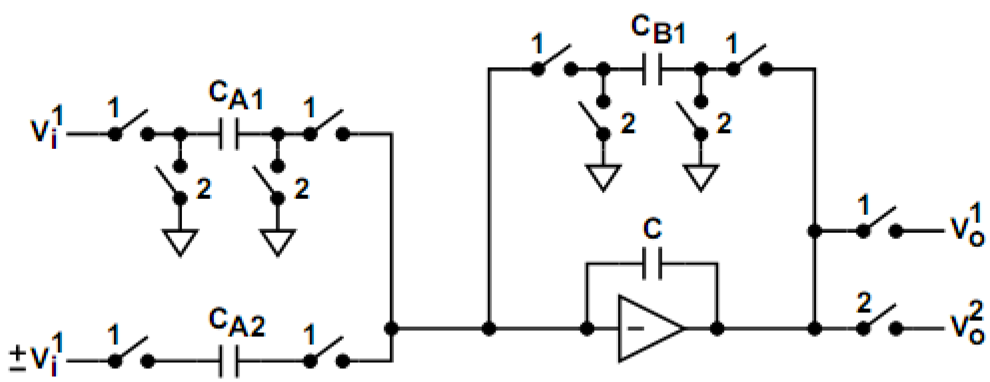
$$V_o^1 = \frac{1}{C\left(1-z^{-1}\right)} \cdot \left[-C_1 V_{i1}^1 + C_2 z^{-1} V_{i2}^1 - C_3\left(1-z^{-1}\right) V_{i3}^1\right]$$

$$V_o^2 = V_o^1 \cdot z^{-1/2}$$

SC 1st-order filter

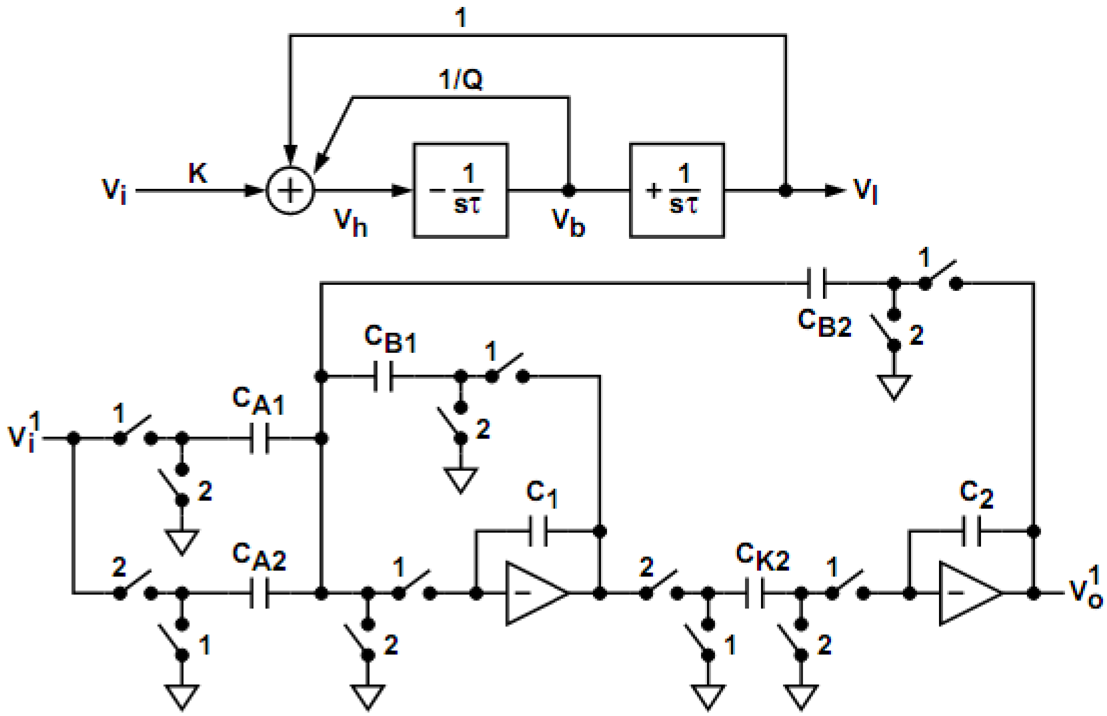






$$\frac{V_o(s)}{V_i(s)} = -\frac{\alpha_1 s + \alpha_0}{s\tau + 1} \qquad \frac{V_o^1}{V_i^1} = -\frac{C_{A1} \pm C_{A2}(1 - z^{-1})}{C_{B1} + C(1 - z^{-1})} = -\frac{\left(\frac{C_{A1}}{C} \pm \frac{C_{A2}}{C}\right) \mp \frac{C_{A2}}{C} z^{-1}}{\left(\frac{C_{B1}}{C} + 1\right) - z^{-1}}$$

SC 2nd-order filter



$$\frac{V_o^1}{V_i^1} = \frac{\left(\frac{C_{A1}}{C_1}\frac{C_{K2}}{C_2}\right) z^{-1} - \left(\frac{C_{A2}}{C_1}\frac{C_{K2}}{C_2}\right) z^{-2}}{\left(\frac{C_{B1}}{C_1} + 1\right) + \left(\frac{C_{B2}}{C_1}\cdot\frac{C_{K2}}{C_2} - \frac{C_{B1}}{C_1} - 2\right) z^{-1} + z^{-2}}$$

SC 2^{nd}-order filter