2. Basic concepts of RFIC design

Nonlinearity

- Harmonic Distortion
- Compression
- Intermodulation
- Dynamic Nonlinear Systems

Noise

- Noise Spectrum
- Device Noise
- Noise in Circuits

Impedance Transformation

- Series–Parallel Conversion
- Matching Networks
- S–Parameters

General considerations: Units in RF design

- Voltage gain and power gain

$$A_V|_{\mathrm{dB}} = 20\log\frac{V_{out}}{V_{in}}$$

$$A_P|_{\mathrm{dB}} = 10\log\frac{P_{out}}{P_{in}}.$$

- Ap and Av are equal if vin and vout appear across equal impedances

$$A_P|_{\mathrm{dB}} = 10\log\frac{\frac{V_{out}^2}{R_0}}{\frac{V_{in}^2}{R_0}} = 20\log\frac{V_{out}}{V_{in}} = A_V|_{\mathrm{dB}},$$

RF power

- dBm: Power is represented in dBm scale
  - log power measured *relative* to 1 mW reference.
  - e.g., a power level of 1 mW = 0 dBm
  - Power level of

$$10.log_{10}\left(\frac{10mW}{1mW}\right) = 10dBm$$

  - In general, $x\ mW$ power can be represented as $10.log_{10}(x)dBm$

- dB is used to represent difference in Power levels
  - e.g.: A power gain of *2* ~ 3 dB gain.

+ Calculation of RF power

Example 2.1

An amplifier senses a sinusoidal signal and delivers a power of 0 dBm to a load resistance of 50 Ω. Determine the peak-to-peak voltage swing across the load.

Solution:

Since 0 dBm is equivalent to 1 mW, for a sinusoidal having a peak-to-peak amplitude of V_{pp} and hence an rms value of $V_{pp}/(2\sqrt{2})$, we write

$$\frac{V_{pp}^2}{8R_L} = 1 \text{ mW}, \tag{2.7}$$

where $R_L = 50\ \Omega$. Thus,

$$V_{pp} = 632 \text{ mV}. \tag{2.8}$$

Calculation of RF power

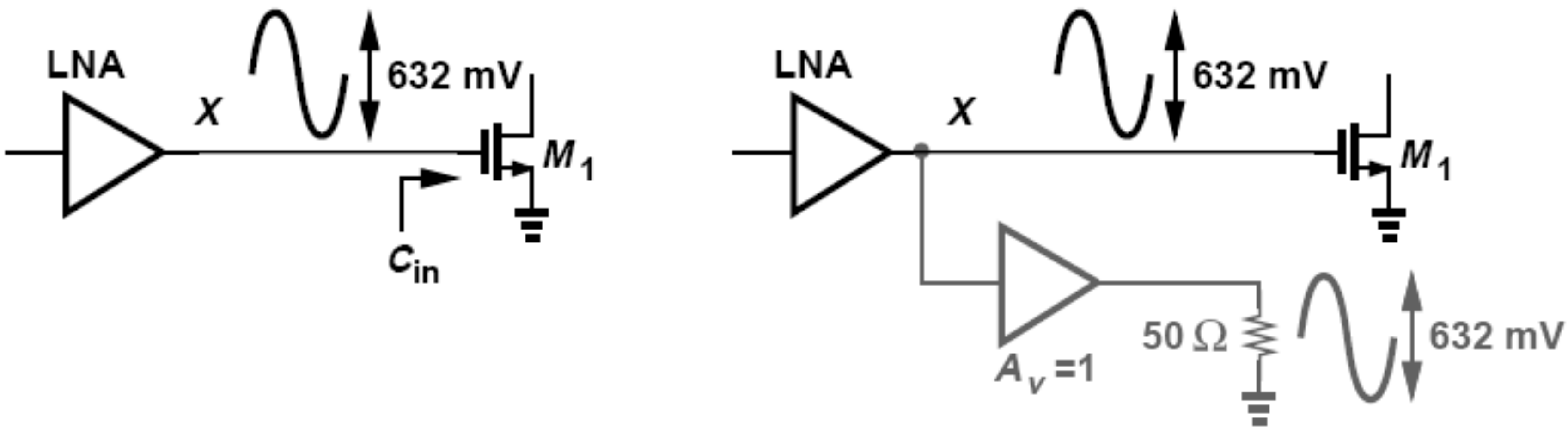
Example 2.2

A GSM receiver senses a narrowband (modulated) signal having a level of -100 dBm. If the front-end amplifier provides a voltage gain of 15 dB, calculate the peak-to-peak voltage swing at the output of the amplifier.

Solution:

Since the amplifier output *voltage* swing is of interest, we first convert the received signal level to voltage. From the previous example, we note that -100 dBm is 100 dB below 632 mV_{pp}. Also, 100 dB for voltage quantities is equivalent to 10^5. Thus, -100 dBm is equivalent to 6.32 μV_{pp}. This input level is amplified by 15 dB ($\approx$ 5.62), resulting in an output swing of 35.5 μV_{pp}.

dBm Used at Interfaces Without Power Transfer



- dBm can be used at interfaces that do not necessarily entail power transfer
- We mentally attach an ideal voltage buffer to node *X* and drive a 50-Ω load. We then say that the signal at node *X* has a level of 0 dBm, tacitly meaning that *if* this signal were applied to a 50-Ω load, *then* it would deliver 1 mW.

Time Variance

- **A system is linear if its output can be expressed as a linear combination (superposition) of responses to individual inputs.**

$$y_1(t) = f[x_1(t)]$$

$$y_2(t) = f[x_2(t)]$$

$$ay_1(t) + by_2(t) = f[ax_1(t) + bx_2(t)].$$

- **A system is time-invariant if a time shift in its input results in the same time shift in its output.**

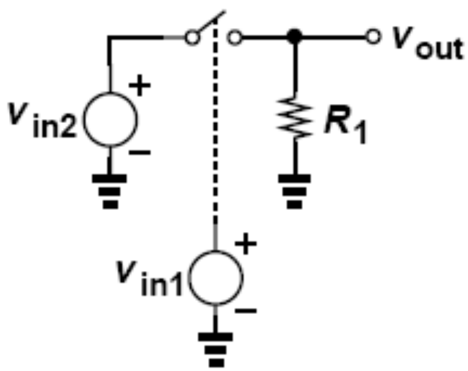
If $y(t) = f[x(t)]$

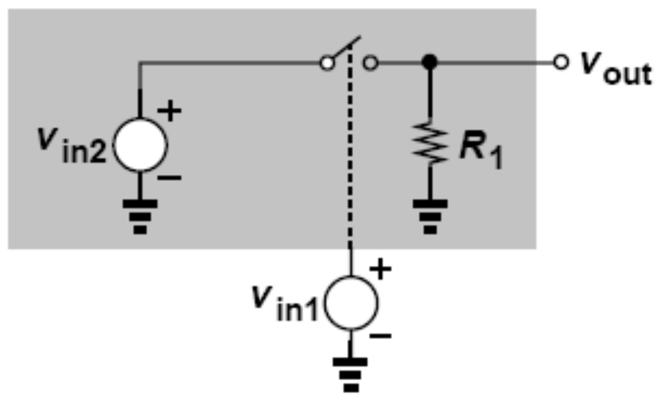
then $y(t-\tau) = f[x(t-\tau)]$

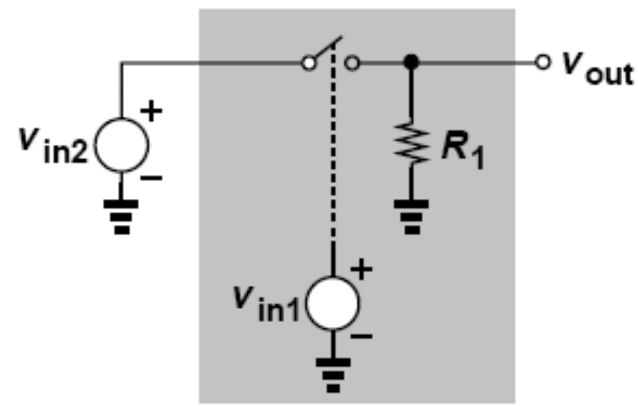
+ Time Variance vs. Nonlinearity

time variance plays a critical role and must not be confused with nonlinearity:

$$v_{in1}(t) = A_1 \cos \omega_1 t \qquad v_{in2}(t) = A_2 \cos \omega_2 t$$







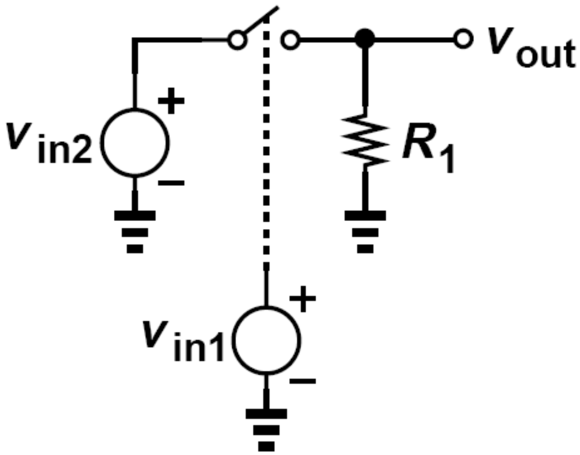
Nonlinear Time Variant

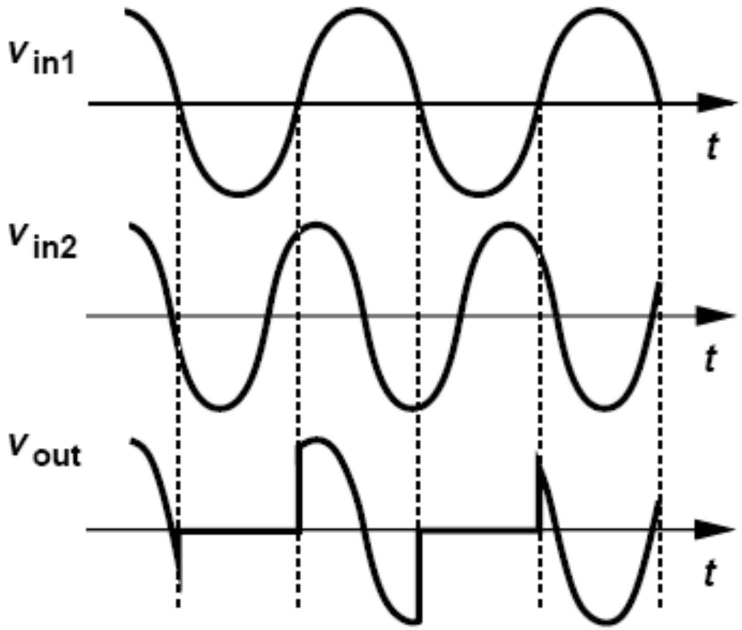
Linear Time Variant

Example of Time Variance

Plot the output waveform of the circuit above if $v_{in1} = A_1 \cos \omega_1 t$ and $v_{in2} = A_2 \cos(1.25\omega_1 t)$.

Solution:





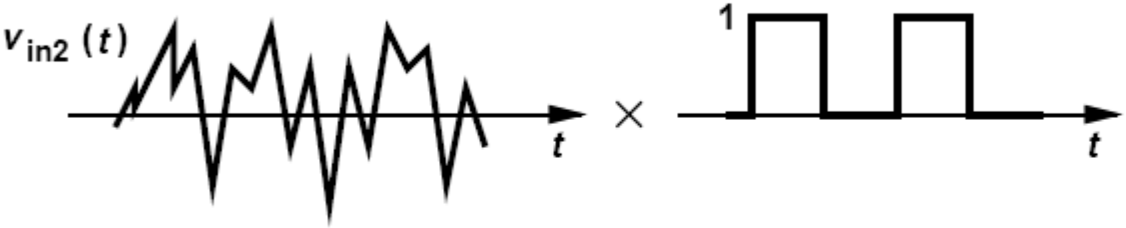
As shown above, v_{out} tracks v_{in2} if $v_{in1} > 0$ and is pulled down to zero by R_1 if $v_{in1} < 0$. That is, v_{out} is equal to the product of v_{in2} and a square wave toggling between 0 and 1.

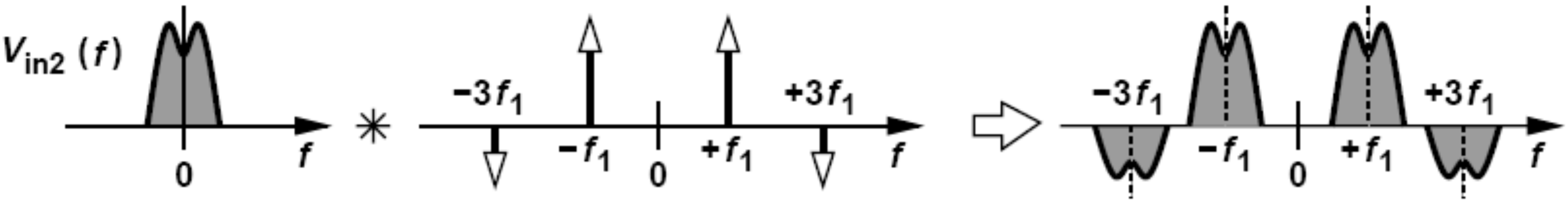
Time Variance: Generation of Other Frequency Components

$$v_{out}(t) = v_{in2}(t) \cdot S(t).$$

$$V_{out}(f) = V_{in2}(f) * \sum_{n=-\infty}^{+\infty} \frac{\sin(n\pi/2)}{n\pi} \delta\left(f - \frac{n}{T_1}\right)$$

$$= \sum_{n=-\infty}^{+\infty} \frac{\sin(n\pi/2)}{n\pi} V_{in2}\left(f - \frac{n}{T_1}\right),$$





- **A linear system can generate frequency components that do not exist in the input signal when system is time variant**

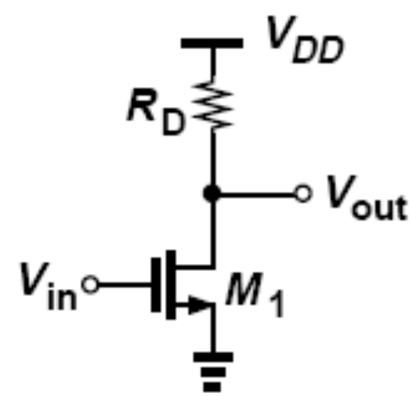
Nonlinearity: Memoryless and Static System

$$y(t) = \alpha x(t),$$ ← **linear**

$$y(t) = \alpha_0 + \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t) + \cdots$$ ← **nonlinear**

- **The input/output characteristic of a memoryless nonlinear system can be approximated with a polynomial**

$$\begin{aligned} V_{out} &= V_{DD} - I_D R_D \\ &= V_{DD} - \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in} - V_{TH})^2 R_D \end{aligned}$$



- **In this idealized case, the circuit displays only second-order nonlinearity**

Example of Polynomial Approximation

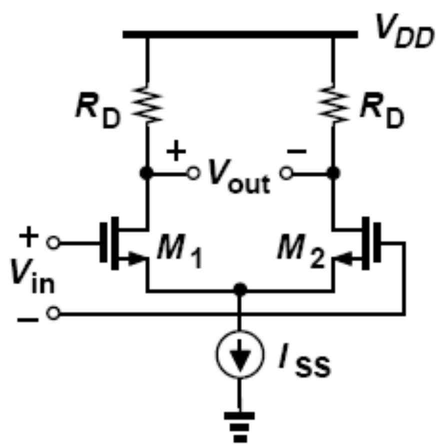
For square-law MOS transistors operating in saturation, the characteristic above can be expressed as

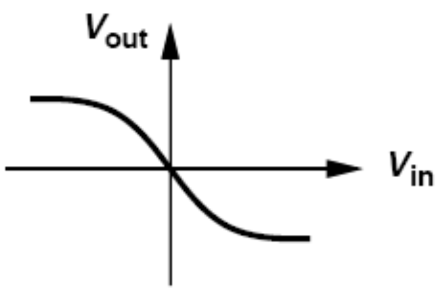
$$V_{out} = -\frac{1}{2}\mu_n C_{ox}\frac{W}{L}V_{in}\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - V_{in}^2}\,R_D$$

If the differential input is small, approximate the characteristic by a polynomial.

Factoring $4I_{ss}/(\mu_n C_{ox} W/L)$ out of the square root and assuming

$$V_{in}^2 \ll \frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}}$$





Approximation gives us:

$$V_{out} \approx -\sqrt{\mu_n C_{ox}\frac{W}{L}I_{SS}V_{in}}\left(1 - \frac{\mu_n C_{ox}\frac{W}{L}}{8I_{SS}}V_{in}^2\right)R_D$$

$$\approx -\sqrt{\mu_n C_{ox}\frac{W}{L}I_{SS}R_D V_{in}} + \frac{\left(\mu_n C_{ox}\frac{W}{L}\right)^{3/2}}{8\sqrt{I_{SS}}}R_D V_{in}^3$$

Effects of Nonlinearity

- Linear system $y(t) = \alpha x(t)$
- Nonlinear system can be approximated by

$$y(t) = \alpha_0 + \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t) + \cdots$$

- Effects of non-linearity
  - Harmonic distortion (HD)
  - Gain compression
  - Cross modulation
  - Intermodulation

Harmonic distortion

$$y(t) \approx \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)$$

$$x(t) = A\cos\omega t$$

$$\begin{aligned} y(t) &= \alpha_1 A\cos\omega t + \alpha_2 A^2\cos^2\omega t + \alpha_3 A^3\cos^3\omega t \\ &= \alpha_1 A\cos\omega t + \frac{\alpha_2 A^2}{2}(1+\cos 2\omega t) + \frac{\alpha_3 A^3}{4}(3\cos\omega t + \cos 3\omega t) \\ &= \frac{\alpha_2 A^2}{2} + \left(\alpha_1 A + \frac{3\alpha_3 A^3}{4}\right)\cos\omega t + \frac{\alpha_2 A^2}{2}\cos 2\omega t + \frac{\alpha_3 A^3}{4}\cos 3\omega t \end{aligned}$$

DC Fundamental HD2 HD3

- Even-order harmonics result from α_j with even j
- nth harmonic grows in proportion to A^n

Total harmonic distortion (THD) = $\dfrac{\text{HD2+HD3+…+HDn}}{\text{Fundamental}}$

Example 2.6

The transmitter in a 900-MHz GSM cellphone delivers 1 W of power to the antenna. Explain the effect of the harmonics of this signal.

Solution:

The second harmonic falls within another GSM cell phone band around 1800 MHz and must be sufficiently small to negligibly impact the other users in that band. The third, fourth, and fifth harmonics do not coincide with any popular bands but must still remain below a certain level imposed by regulatory organizations in each country. The sixth harmonic falls in the 5-GHz band used in wireless local area networks (WLANs), e.g., in laptops. Figure 2.8 summarizes these results.

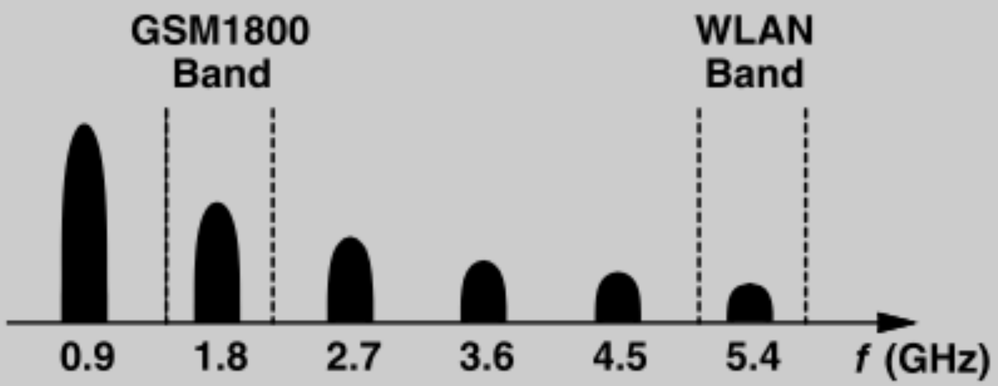


Figure 2.8 *Summary of harmonic components.*

Example 2.5

An analog multiplier "mixes" its two inputs as shown in Fig. 2.7, ideally producing $y(t) = kx_1(t)x_2(t)$, where k is a constant.[3] Assume $x_1(t) = A_1 \cos \omega_1 t$ and $x_2(t) = A_2 \cos \omega_2 t$.

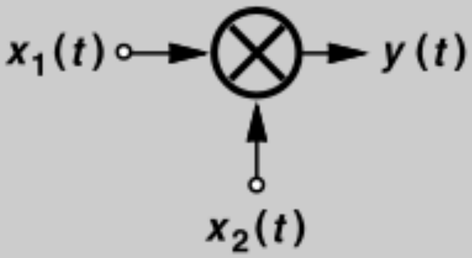


Figure 2.7 *Analog multiplier.*

(a) If the mixer is ideal, determine the output frequency components.
(b) If the input port sensing $x_2(t)$ suffers from third-order nonlinearity, determine the output frequency components.

Solution:

(a) We have

$$y(t) = k(A_1 \cos \omega_1 t)(A_2 \cos \omega_2 t) \tag{2.29}$$

$$= \frac{kA_1A_2}{2} \cos(\omega_1 + \omega_2)t + \frac{kA_1A_2}{2} \cos(\omega_1 - \omega_2)t. \tag{2.30}$$

The output thus contains the sum and difference frequencies. These may be considered "desired" components.

(b) Representing the third harmonic of $x_2(t)$ by $(\alpha_3 A_2^3/4) \cos 3\omega_2 t$, we write

$$y(t) = k(A_1 \cos \omega_1 t)\left(A_2 \cos \omega_2 t + \frac{\alpha_3 A_2^3}{4} \cos 3\omega_2 t\right) \tag{2.31}$$

$$= \frac{kA_1A_2}{2} \cos(\omega_1 + \omega_2)t + \frac{kA_1A_2}{2} \cos(\omega_1 - \omega_2)t$$
$$+ \frac{k\alpha_3 A_1 A_2^3}{8} \cos(\omega_1 + 3\omega_2)t + \frac{k\alpha_3 A_1 A_2^3}{8} \cos(\omega_1 - 3\omega_2)t. \tag{2.32}$$

The mixer now produces two "spurious" components at $\omega_1 + 3\omega_2$ and $\omega_1 - 3\omega_2$, one or both of which often prove problematic. For example, if $\omega_1 = 2\pi \times (850 \text{ MHz})$ and $\omega_2 = 2\pi \times (900 \text{ MHz})$, then $|\omega_1 - 3\omega_2| = 2\pi \times (1850 \text{ MHz})$, an "undesired" component that is difficult to filter because it lies close to the desired component at $\omega_1 + \omega_2 = 2\pi \times (1750 \text{ MHz})$.

Gain compression

$$y(t) = \alpha_1 A \cos \omega t + \alpha_2 A^2 \cos^2 \omega t + \alpha_3 A^3 \cos^3 \omega t$$

$$= \alpha_1 A \cos \omega t + \frac{\alpha_2 A^2}{2}(1 + \cos 2\omega t) + \frac{\alpha_3 A^3}{4}(3 \cos \omega t + \cos 3\omega t)$$

$$= \frac{\alpha_2 A^2}{2} + \left(\alpha_1 A + \frac{3\alpha_3 A^3}{4}\right) \cos \omega t + \frac{\alpha_2 A^2}{2} \cos 2\omega t + \frac{\alpha_3 A^3}{4} \cos 3\omega t$$

Typically $\alpha_1 \alpha_3 < 0$

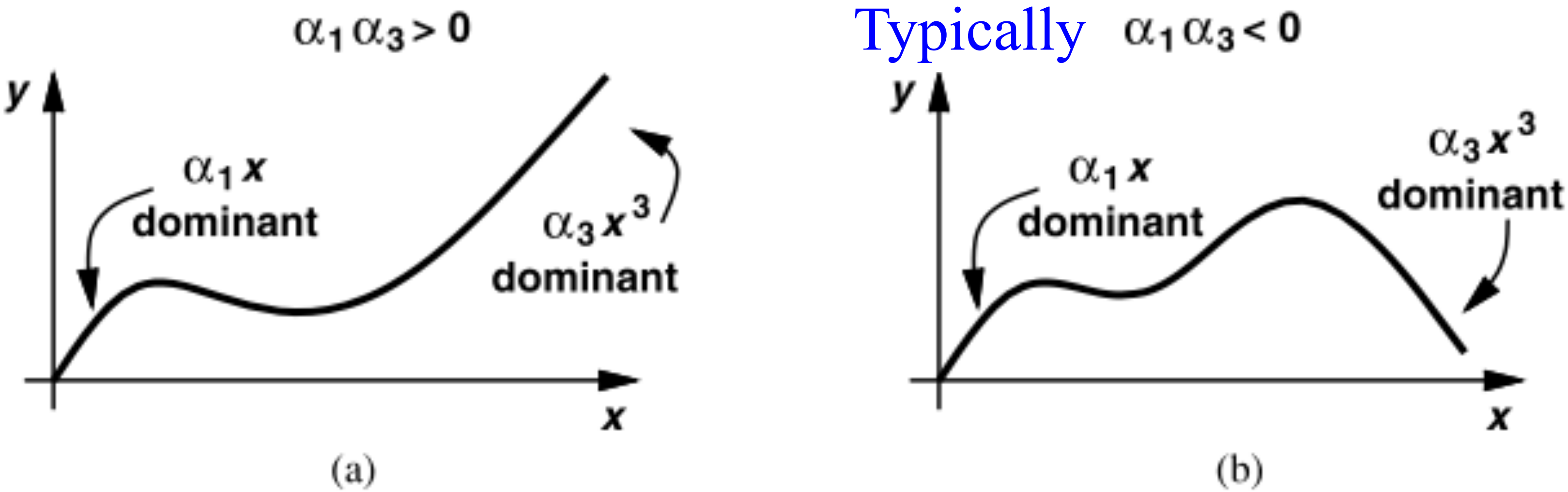
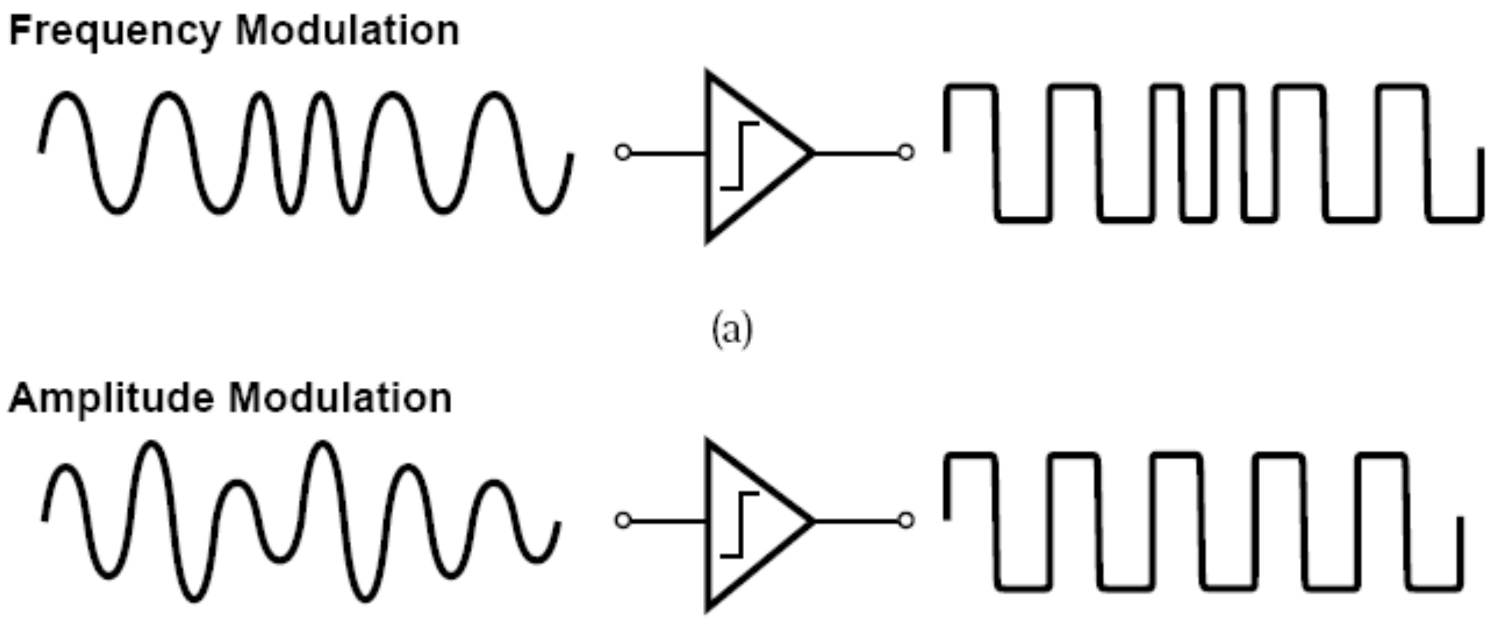


Figure 2.9 *(a) Expansive and (b) compressive characteristics.*

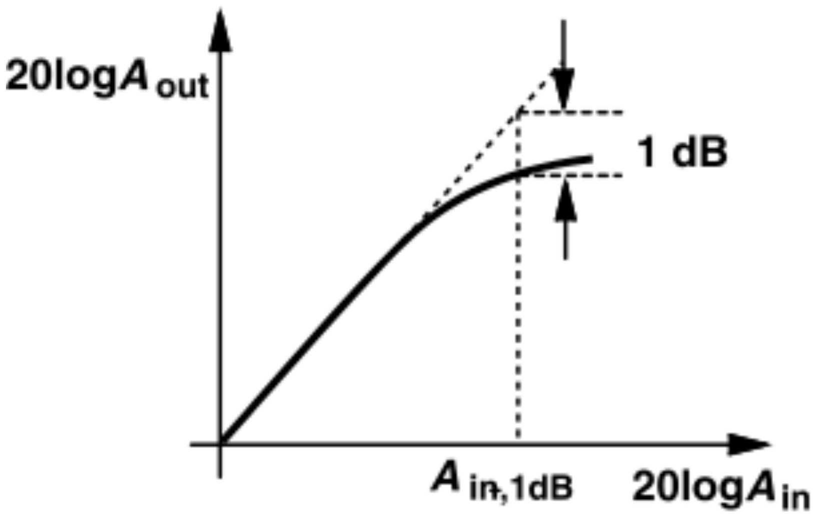
Gain Compression: Effect on FM and AM Waveforms



- FM signal carries no information in its amplitude and hence tolerates compression.
- AM contains information in its amplitude, hence distorted by compression

1-dB compression point

- Gain depends on input amplitudes



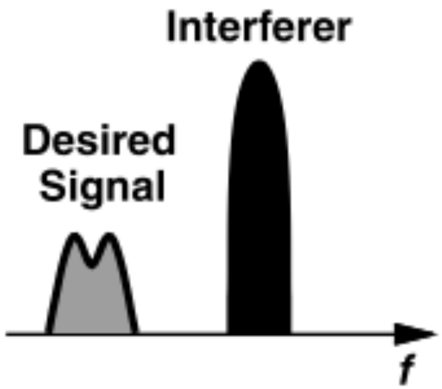
$$20\log\left|\alpha_1+\frac{3}{4}\alpha_3A_{in,1dB}^2\right|=20\log|\alpha_1|-1\text{ dB}.$$

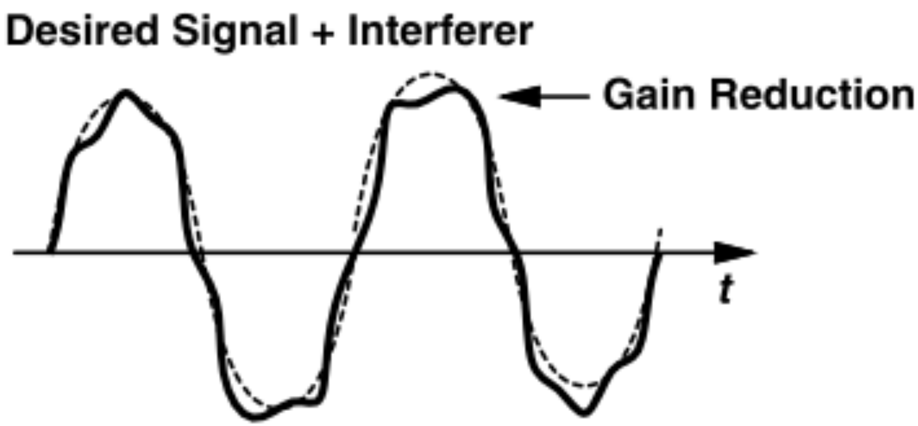
$$A_{in,1dB}=\sqrt{0.145\left|\frac{\alpha_1}{\alpha_3}\right|}.$$

- Typically Pin-1dB is about -20 to -25 dBm

Desensitization

- If a weak signal and a strong interferer experience a compressive nonlinearity, the average gain for the weak signal decreases. We say the large interference desensitizes the circuits.





$$x(t) = A_1 \cos\omega_1 t + A_2 \cos\omega_2 t$$

$$y(t) = \left(\alpha_1 + \frac{3}{4}\alpha_3 A_1^2 + \frac{3}{2}\alpha_3 A_2^2\right) A_1 \cos\omega_1 t + \cdots$$

$$\text{If} \quad A_1 << A_2 : y(t) = \left(\alpha_1 A_1 + \frac{3}{2}\alpha_3 A_1 A_2^2\right)\cos\omega_1 t + \cdots$$

Gain can drop to zero, i.e. signal is blocked

Example 2.7

A 900-MHz GSM transmitter delivers a power of 1 W to the antenna. By how much must the second harmonic of the signal be suppressed (filtered) so that it does not desensitize a 1.8-GHz receiver having $P_{1dB} = -25\,\text{dBm}$? Assume the receiver is 1 m away (Fig. 2.13) and the 1.8-GHz signal is attenuated by 10 dB as it propagates across this distance.

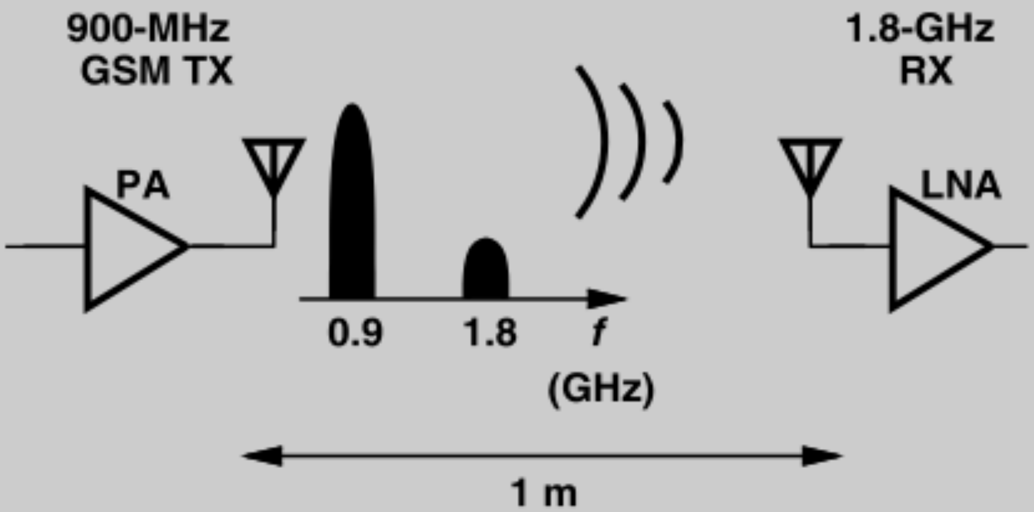


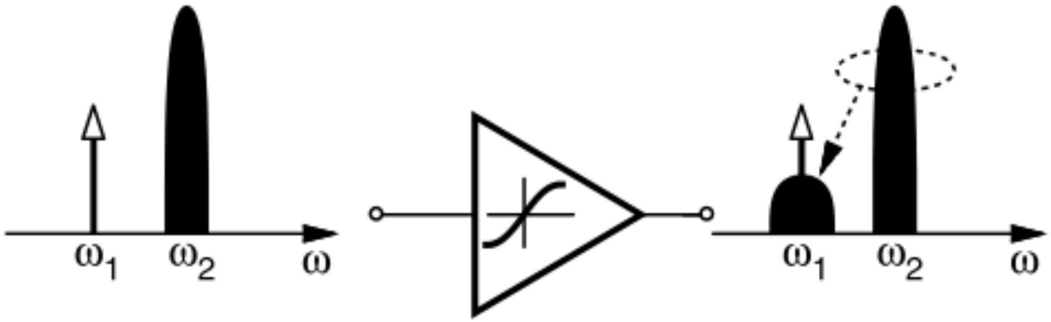
Figure 2.13 *TX and RX in a cellular system.*

Solution:

The output power at 900 MHz is equal to +30 dBm. With an attenuation of 10 dB, the second harmonic must not exceed −15 dBm at the transmitter antenna so that it is below P_{1dB} of the receiver. Thus, the second harmonic must remain at least 45 dB below the fundamental at the TX output. In practice, this interference must be another several dB lower to ensure the RX does not compress.

Cross Modulation

Transfer of modulation on the amplitude of the interferer to the amplitude of the weak signal.



When a weak signal and a strong interferer pass through a nonlinear system,

Weak signal: $x_1(t) = A_1 \cos \omega_1 t$

Strong interferer: $x_2(t) = A_2 (1 + m \cos \omega_m t) \cos \omega_2 t$

Then,

$$y(t) = \left[\alpha_1 + \frac{3}{2}\alpha_3 A_2^2 \left(1 + \frac{m^2}{2} + \frac{m^2}{2}\cos 2\omega_m t + 2m \cos \omega_m t\right)\right] A_1 \cos \omega_1 t + \cdots$$

Example 2.8

Suppose an interferer contains phase modulation but not amplitude modulation. Does cross modulation occur in this case?

Solution:

Expressing the input as $x(t) = A_1 \cos \omega_1 t + A_2 \cos(\omega_2 t + \phi)$, where the second term represents the interferer (A_2 is constant but ϕ varies with time), we use the third-order polynomial in Eq. (2.25) to write

$$y(t) = \alpha_1[A_1 \cos \omega_1 t + A_2 \cos(\omega_2 t + \phi)] + \alpha_2[A_1 \cos \omega_1 t + A_2 \cos(\omega_2 t + \phi)]^2 + \alpha_3[A_1 \cos \omega_1 t + A_2 \cos(\omega_2 t + \phi)]^3. \tag{2.38}$$

We now note that (1) the second-order term yields components at $\omega_1 \pm \omega_2$ but not at ω_1; (2) the third-order term expansion gives $3\alpha_3 A_1 \cos \omega_1 t A_2^2 \cos^2(\omega_2 t + \phi)$, which, according to $\cos^2 x = (1 + \cos 2x)/2$, results in a component at ω_1. Thus,

$$y(t) = \left(\alpha_1 + \frac{3}{2}\alpha_3 A_2^2\right) A_1 \cos \omega_1 t + \cdots . \tag{2.39}$$

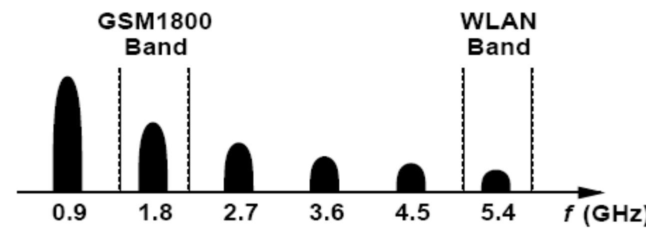
Interestingly, the desired signal at ω_1 does not experience cross modulation. That is, phase-modulated interferers do not cause cross modulation in *memoryless* (static) nonlinear systems. Dynamic nonlinear systems, on the other hand, may not follow this rule.

Intermodulation

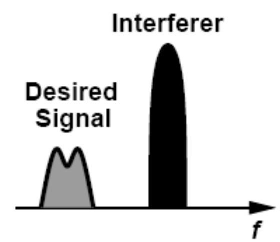
So far we have considered the case of:

- **Single Signal**
- **Signal + one large interferer**
- **Signal + two large interferers**

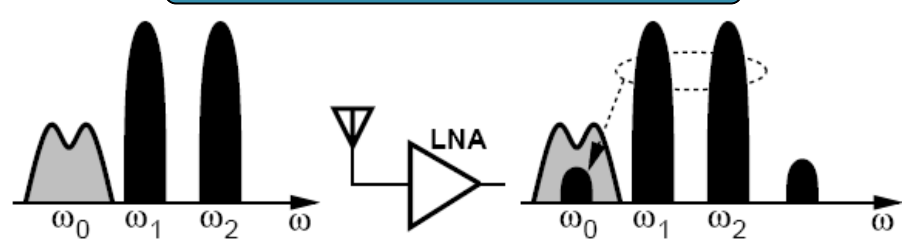
Harmonic distortion



Desensitization



Intermodulation



+ Intermodulation

$$x(t) = A_1 \cos\omega_1 t + A_2 \cos\omega_2 t \qquad y(t) = \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t) + \cdots$$

$$y(t) = \alpha_1(A_1 \cos\omega_1 t + A_2 \cos\omega_2 t) + \alpha_2(A_1 \cos\omega_1 t + A_2 \cos\omega_2 t)^2 + \alpha_3(A_1 \cos\omega_1 t + A_2 \cos\omega_2 t)^3.$$

Fundamental products

$$\omega = \omega_1,\ \omega_2 : \left(\alpha_1 A_1 + \frac{3}{4}\alpha_3 A_1^3 + \frac{3}{2}\alpha_3 A_1 A_2^2\right)\cos\omega_1 t + \left(\alpha_1 A_2 + \frac{3}{4}\alpha_3 A_2^3 + \frac{3}{2}\alpha_3 A_2 A_1^2\right)\cos\omega_2 t$$

2nd-order intermodulation Products (IM2)

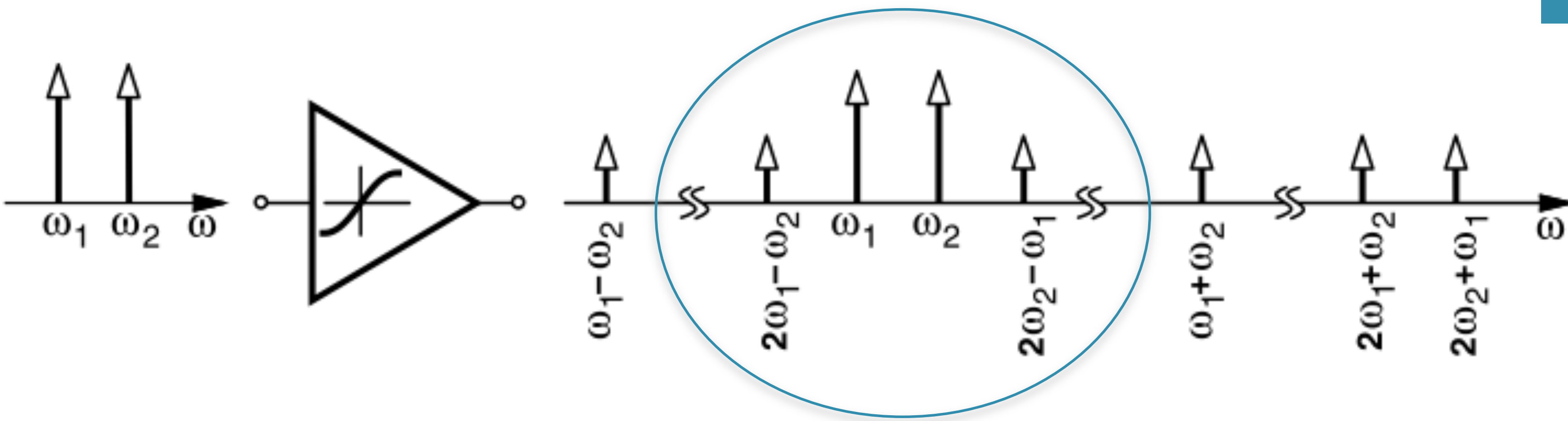
$$\omega = \omega_1 \pm \omega_2 : \alpha_2 A_1 A_2 \cos(\omega_1 + \omega_2)t + \alpha_2 A_1 A_2 \cos(\omega_1 - \omega_2)t$$

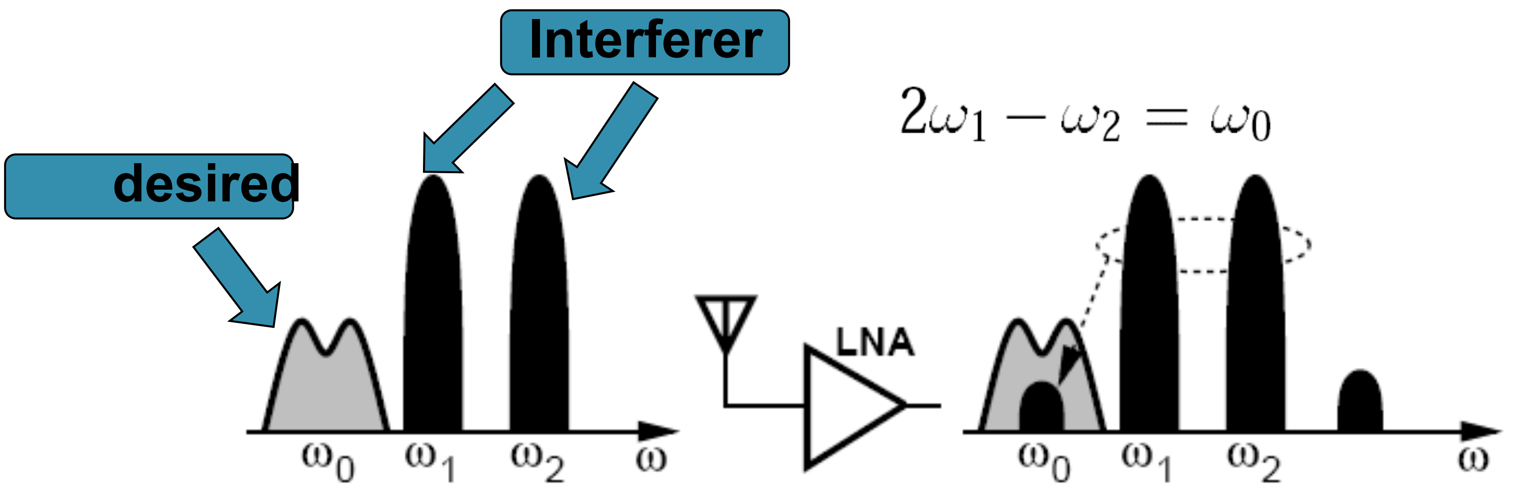
3rd-order Intermodulation Products (IM3)

$$\omega = 2\omega_1 \pm \omega_2 : \frac{3\alpha_3 A_1^2 A_2}{4}\cos(2\omega_1 + \omega_2)t + \frac{3\alpha_3 A_1^2 A_2}{4}\cos(2\omega_1 - \omega_2)t$$

$$\omega = 2\omega_2 \pm \omega_1 : \frac{3\alpha_3 A_1 A_2^2}{4}\cos(2\omega_2 + \omega_1)t + \frac{3\alpha_3 A_1 A_2^2}{4}\cos(2\omega_2 - \omega_1)t$$

+ Intermodulation





- A received small desired signal along with two large interferers
- Intermodulation product falls onto the desired channel, corrupts signal.

Example 2.9

Suppose four Bluetooth users operate in a room as shown in Fig. 2.17. User 4 is in the receive mode and attempts to sense a weak signal transmitted by User 1 at 2.410 GHz.

At the same time, Users 2 and 3 transmit at 2.420 GHz and 2.430 GHz, respectively. Explain what happens.

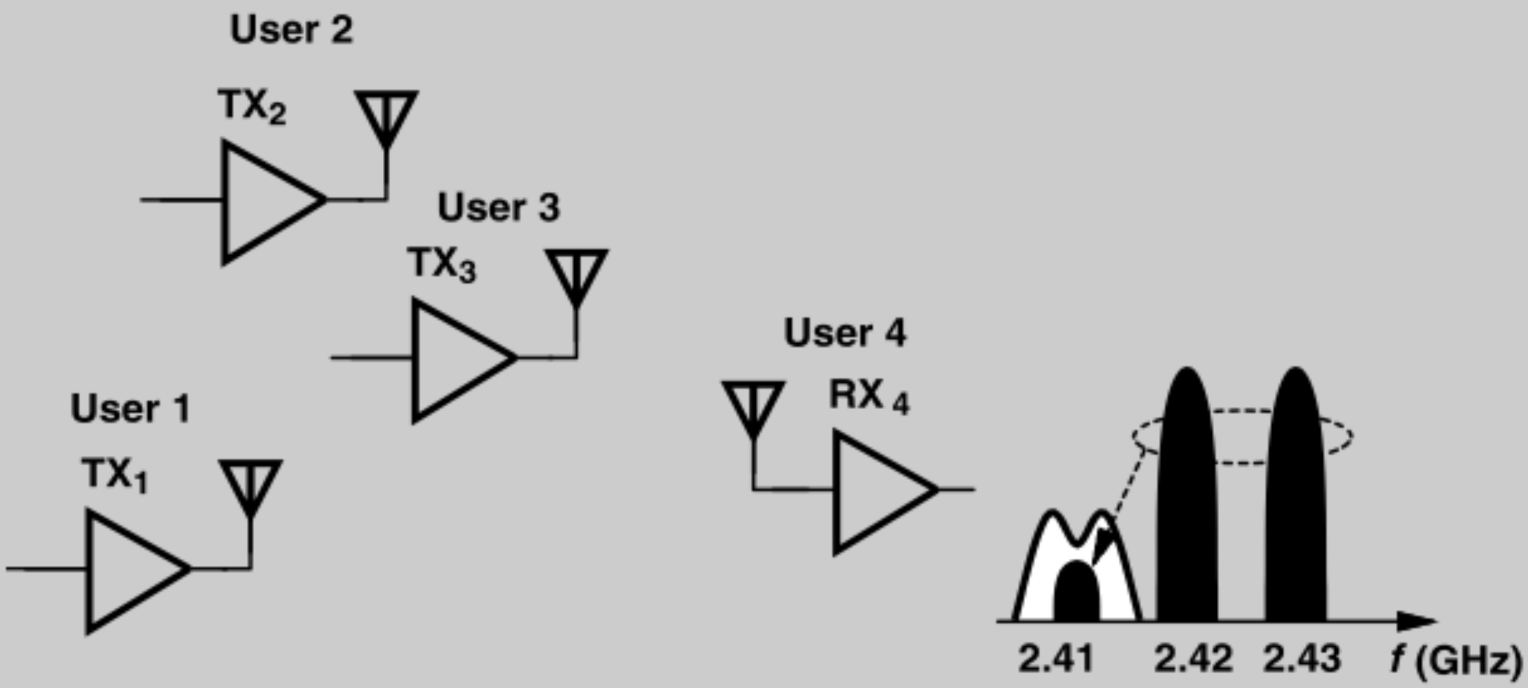


Figure 2.17 *Bluetooth RX in the presence of several transmitters.*

Solution:

Since the frequencies transmitted by Users 1, 2, and 3 happen to be equally spaced, the intermodulation in the LNA of RX_4 corrupts the desired signal at 2.410 GHz.

Example 2.10

A Bluetooth receiver employs a low-noise amplifier having a gain of 10 and an input impedance of 50 Ω. The LNA senses a desired signal level of −80 dBm at 2.410 GHz and two interferers of equal levels at 2.420 GHz and 2.430 GHz. For simplicity, assume the LNA drives a 50-Ω load.

(a) Determine the value of α_3 that yields a P_{1dB} of −30 dBm.

(b) If each interferer is 10 dB below P_{1dB}, determine the corruption experienced by the desired signal at the LNA output.

Solution:

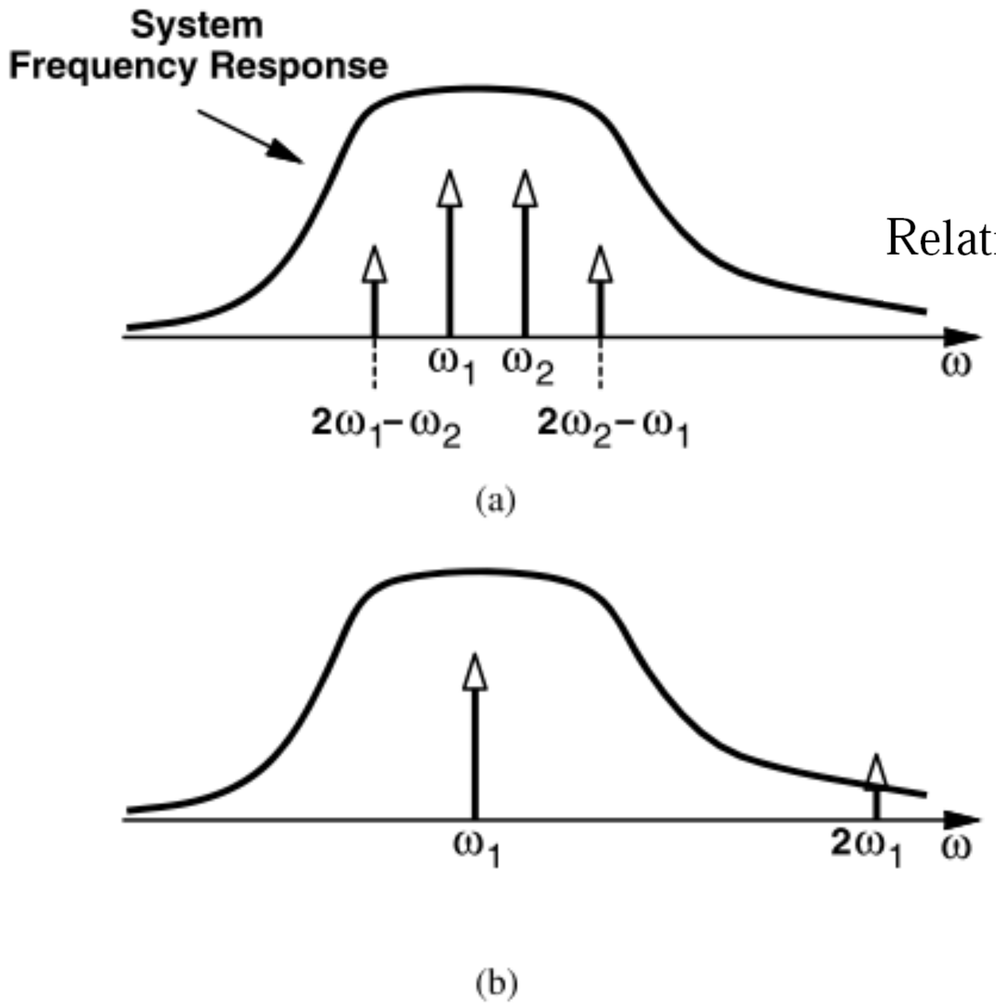
(a) Noting that $-30\,\text{dBm} = 20\,\text{mV}_{\text{pp}} = 10\,\text{mV}_{\text{p}}$, from Eq. (2.34), we have $\sqrt{0.145|\alpha_1/\alpha_3|} = 10\,\text{mV}_{\text{p}}$. Since $\alpha_1 = 10$, we obtain $\alpha_3 = 14,500\,\text{V}^{-2}$.

(b) Each interferer has a level of $-40\,\text{dBm}$ $(= 6.32\,\text{mV}_{\text{pp}})$. Setting $A_1 = A_2 = 6.32\,\text{mV}_{\text{pp}}/2$ in Eq. (2.41), we determine the amplitude of the IM product at 2.410 GHz as

$$\frac{3\alpha_3 A_1^2 A_2}{4} = 0.343\,\text{mV}_{\text{p}} = -59.3\,\text{dBm}. \tag{2.44}$$

The desired signal is amplified by a factor of $\alpha_1 = 10 = 20\,\text{dB}$, emerging at the output at a level of −60 dBm. Unfortunately, the IM product is as large as the signal itself even though the LNA does not experience significant compression.

IMD vs HD for narrowband system



$$\text{Relative IM} = 20\log\left(\frac{3}{4}\frac{\alpha_3}{\alpha_1}A^2\right)\ \text{dBc}$$

If the input sinusoid frequency is chosen such that its harmonics fall out of the passband,
The output distortion appears quite small even if the input stage of the filter introduces substantial nonlinearity.
In many cases, harmonic distortion is not adequate to characterize the non-linearity.

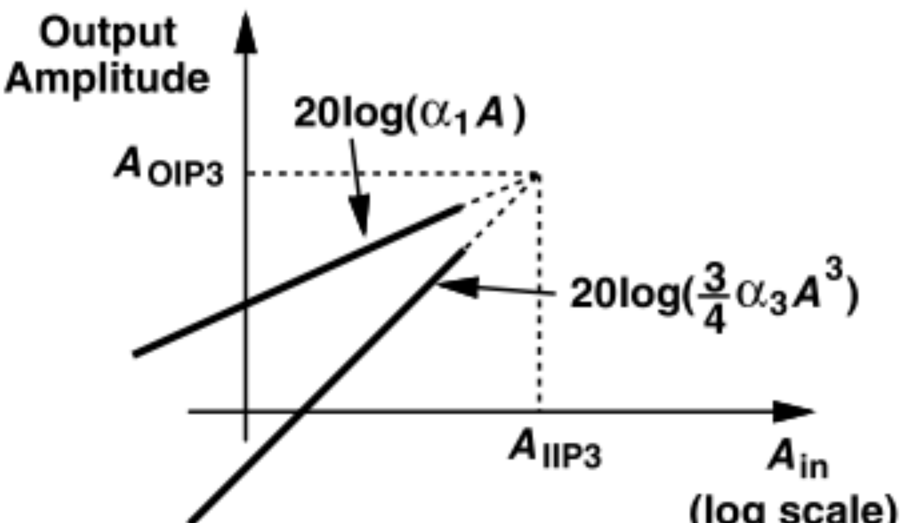
Figure 2.19 *(a) Two-tone and (b) harmonic tests in a narrowband system.*

Third-order intercept point (IP_3)

- Using two tones with the same amplitude, we increase the input level. The fundamentals at the output increases in proportion to *A* whereas the IM products increase in proportion to A^3.

$$y(t)=\left(\alpha_1+\frac{3}{2}\alpha_3A^2\right)A\cos\omega_1 t+\left(\alpha_1+\frac{3}{2}\alpha_3A^2\right)A\cos\omega_2 t+$$

$$\frac{3}{4}\alpha_3A^3\cos(2\omega_1-\omega_2)t+\frac{3}{4}\alpha_3A^3\cos(2\omega_2-\omega_1)t+\cdots$$



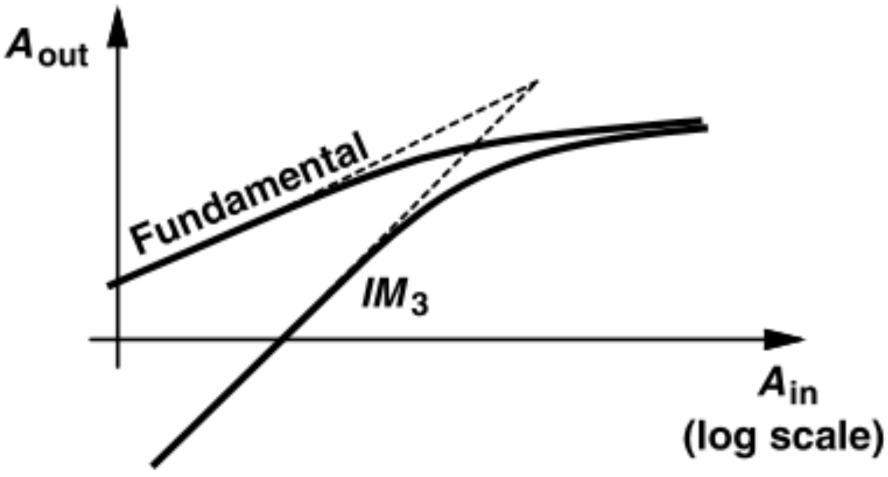
$$|\alpha_1 A_{IIP3}| = \left|\frac{3}{4}\alpha_3 A_{IIP3}^3\right|,$$

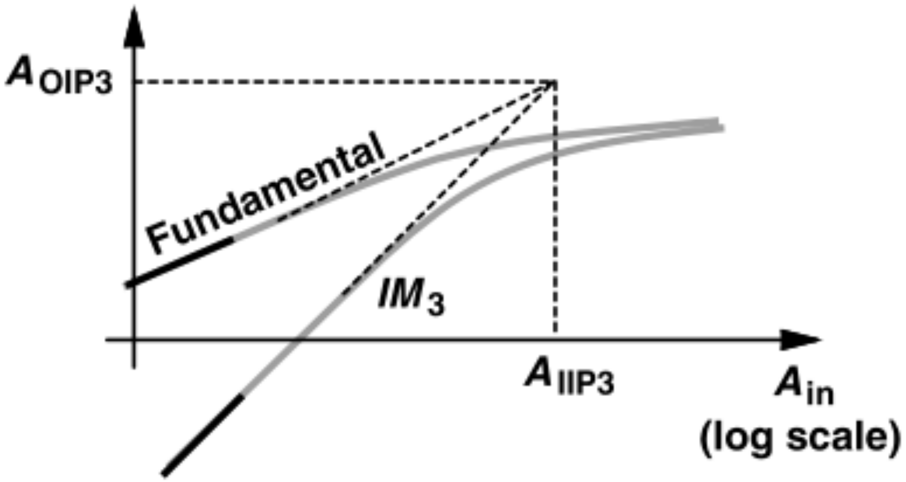
$$A_{IIP3} = \sqrt{\frac{4}{3}\left|\frac{\alpha_1}{\alpha_3}\right|}.$$

$$\frac{A_{IIP3}}{A_{1dB}} = \sqrt{\frac{4}{0.435}}$$

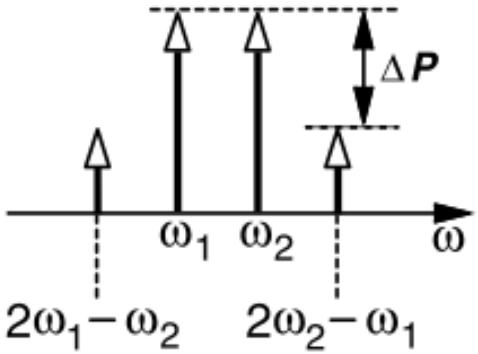
$$\approx 9.6\,\text{dB}.$$

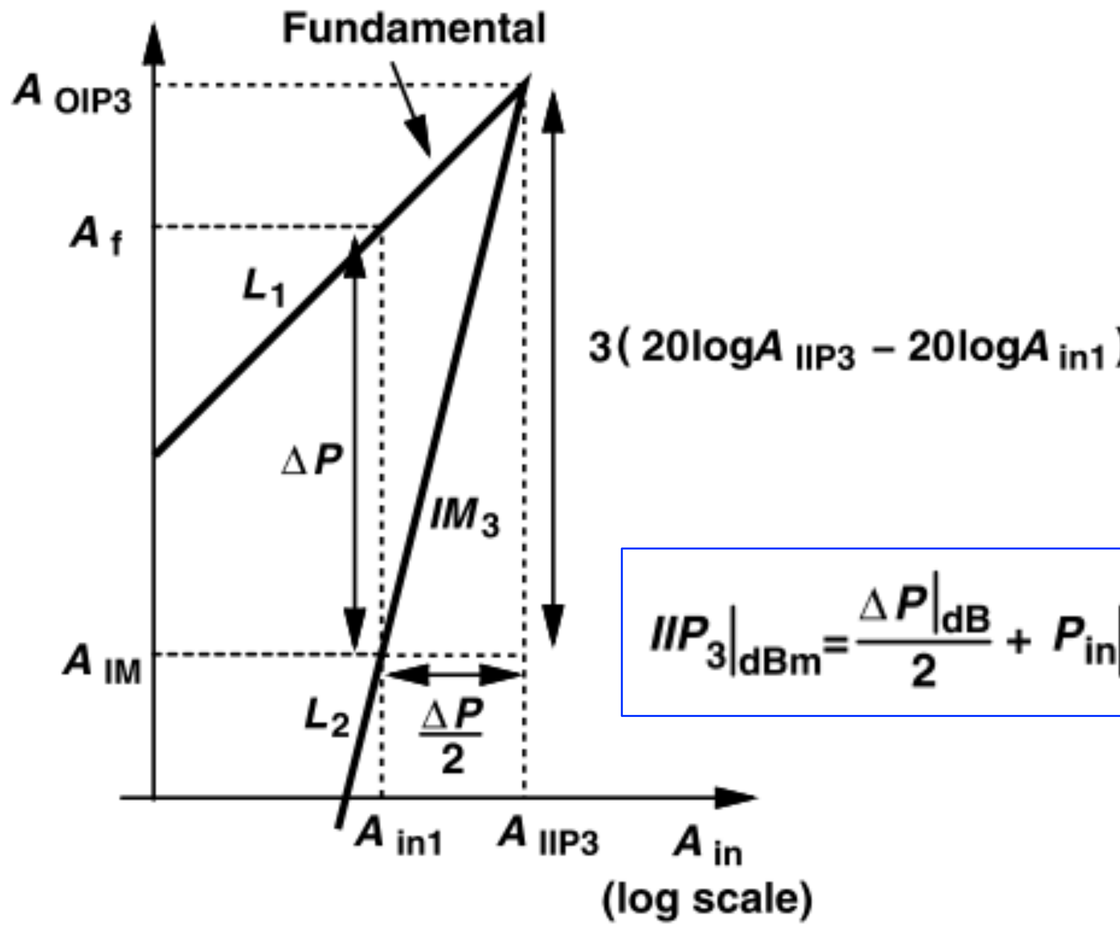
IP3 calculation from measurement





IP3 Estimation





$$IIP_3\big|_{dBm} = \frac{\Delta P\big|_{dB}}{2} + P_{in}\big|_{dBm}$$

$$\Delta P = 20\log A_f - 20\log A_{IM} = 2(20\log A_{IIP3} - 20\log A_{in1})$$

$$20\log A_{IIP3} = \frac{\Delta P}{2} + 20\log A_{in1}$$

Example 2.11

A low-noise amplifier senses a −80-dBm signal at 2.410 GHz and two −20-dBm interferers at 2.420 GHz and 2.430 GHz. What IIP_3 is required if the IM products must remain 20 dB below the signal? For simplicity, assume 50-Ω interfaces at the input and output.

Solution:

Denoting the peak amplitudes of the signal and the interferers by A_{sig} and A_{int}, respectively, we can write at the LNA output:

$$20\log|\alpha_1 A_{sig}| - 20\,\text{dB} = 20\log\left|\frac{3}{4}\alpha_3 A_{int}^3\right|. \tag{2.50}$$

It follows that

$$|\alpha_1 A_{sig}| = \left|\frac{30}{4}\alpha_3 A_{int}^3\right|. \tag{2.51}$$

In a 50-Ω system, the −80-dBm and −20-dBm levels respectively yield $A_{sig} = 31.6\,\mu V_p$ and $A_{int} = 31.6\,mV_p$. Thus,

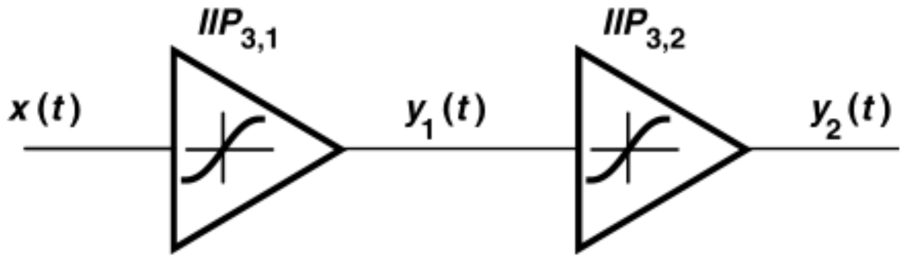
$$IIP_3 = \sqrt{\frac{4}{3}\left|\frac{\alpha_1}{\alpha_3}\right|} \tag{2.52}$$

$$= 3.65\,V_p \tag{2.53}$$

$$= +15.2\,\text{dBm}. \tag{2.54}$$

Such an IP_3 is extremely difficult to achieve, especially for a complete receiver chain.

Intermodulation in cascade stages



$$y_1(t) = \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)$$

$$y_2(t) = \beta_1 y_1(t) + \beta_2 y_1^2(t) + \beta_3 y_1^3(t).$$

$$y_2(t) = \beta_1[\alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)] + \beta_2[\alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)]^2 + \beta_3[\alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)]^3.$$

$$y_2(t) = \alpha_1\beta_1 x(t) + (\alpha_3\beta_1 + 2\alpha_1\alpha_2\beta_2 + \alpha_1^3\beta_3)x^3(t) + \cdots .$$

$$A_{IP3} = \sqrt{\frac{4}{3}\left|\frac{\alpha_1\beta_1}{\alpha_3\beta_1 + 2\alpha_1\alpha_2\beta_2 + \alpha_1^3\beta_3}\right|}.$$

Intermodulation in cascade stages

$$\frac{1}{A_{IP3}^2} = \frac{3}{4}\left|\frac{\alpha_3\beta_1 + 2\alpha_1\alpha_2\beta_2 + \alpha_1^3\beta_3}{\alpha_1\beta_1}\right|$$

$$= \frac{3}{4}\left|\frac{\alpha_3}{\alpha_1} + \frac{2\alpha_2\beta_2}{\beta_1} + \frac{\alpha_1^2\beta_3}{\beta_1}\right|$$

$$= \left|\frac{1}{A_{IP3,1}^2} + \frac{3\alpha_2\beta_2}{2\beta_1} + \frac{\alpha_1^2}{A_{IP3,2}^2}\right|,$$

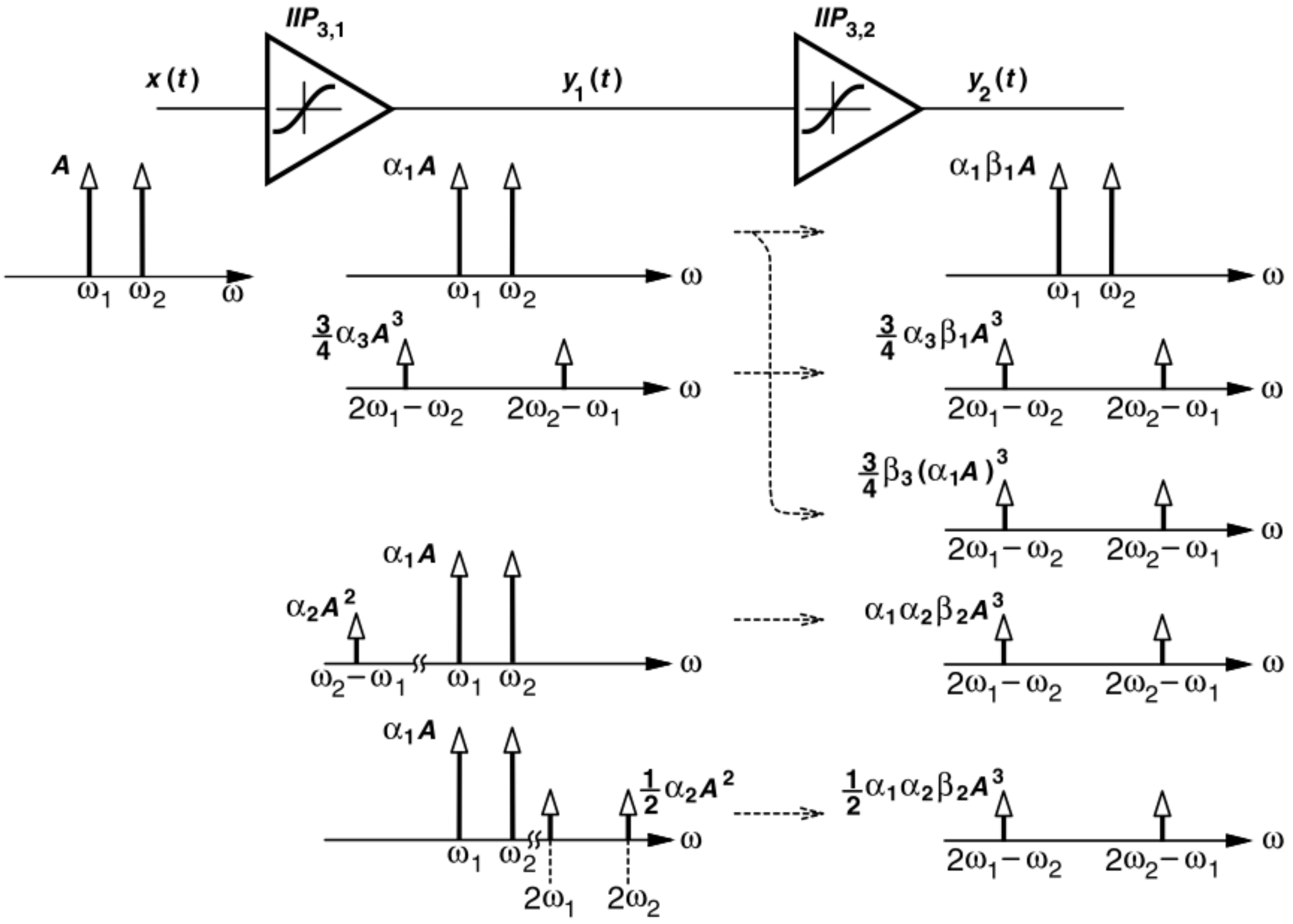
$$\frac{1}{A_{IP3}^2} \approx \frac{1}{A_{IP3,1}^2} + \frac{\alpha_1^2}{A_{IP3,2}^2}.$$

The higher the gain of the 1st stage, the more nonlinearity of the 2nd stage

$$\frac{1}{A_{IP3}^2} \approx \frac{1}{A_{IP3,1}^2} + \frac{\alpha_1^2}{A_{IP3,2}^2} + \frac{\alpha_1^2\beta_1^2}{A_{IP3,3}^2} + \cdots$$

- Thus, if each stage in a cascade has a gain greater than unity, the nonlinearity of the latter stages becomes increasingly more critical because the IP3 of each stage is equivalently scaled down by the total gain preceding that stage.

Intermodulation in cascade stages



Example of Cascaded Nonlinear Stages

A low-noise amplifier having an input IP_3 of -10 dBm and a gain of 20 dB is followed by a mixer with an input IP_3 of +4 dBm. Which stage limits the IP_3 of the cascade more?

Solution:

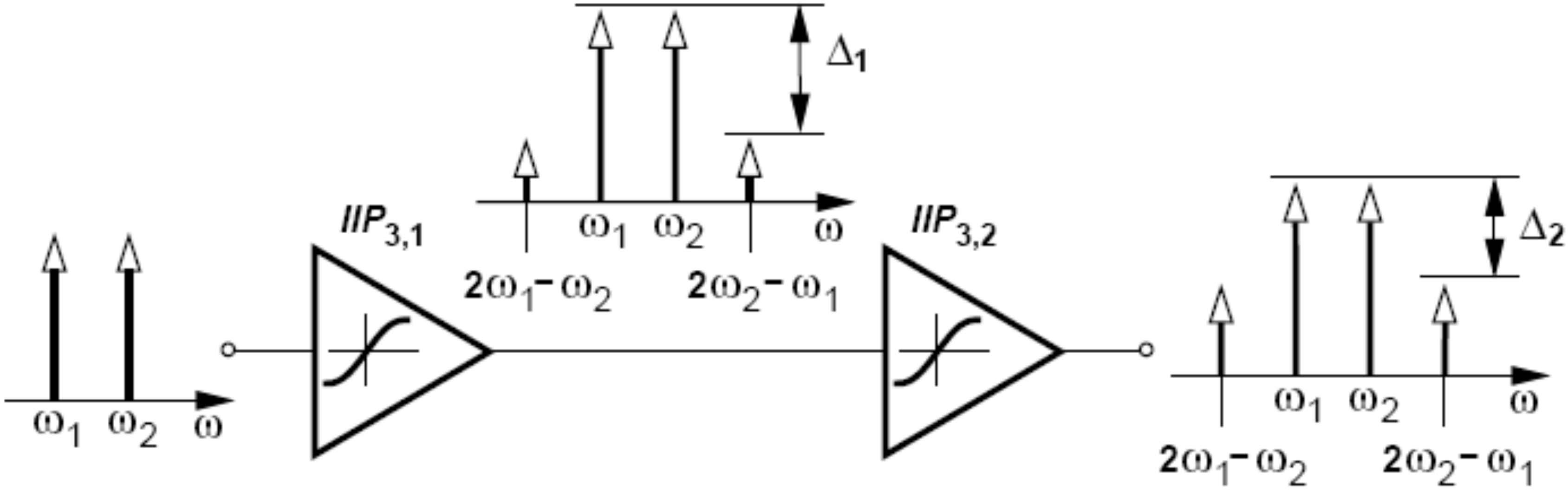
With α_1 = 20 dB, we note that

$$A_{IP3,1} = -10\ \text{dBm}$$

$$\frac{A_{IP3,2}}{\alpha_1} = -16\ \text{dBm}$$

Since the scaled IP_3 of the second stage is lower than the IP_3 of the first stage, we say the second stage limits the overall IP_3 more.

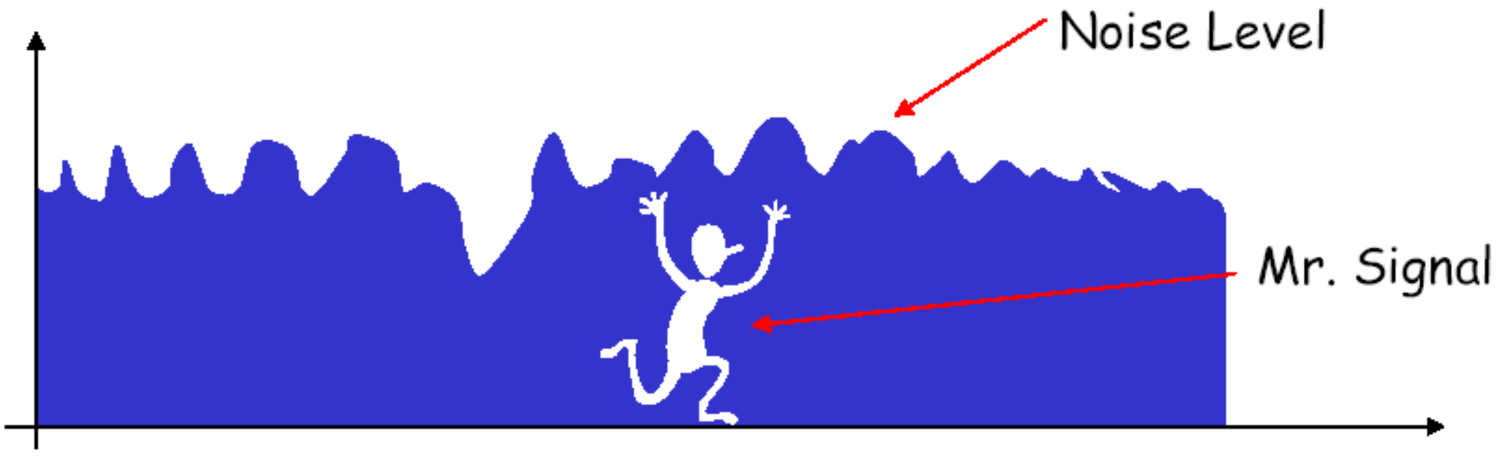
Linearity Limit due to Each Stage



- **Examine the relative IM magnitudes at the output of each stage to find out which stage limits the linearity more**

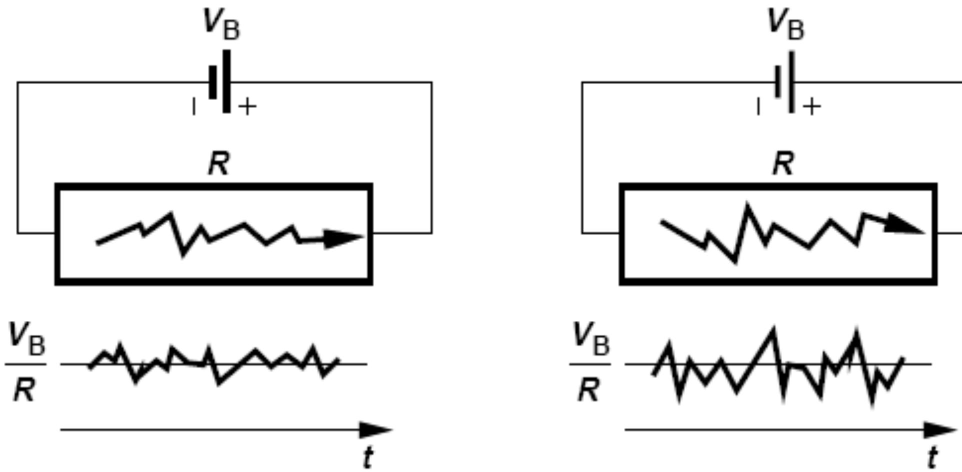
Noise

- สัญญาณรบกวน กำหนดระดับของสัญญาณที่ต่ำที่สุด ที่วงจรสามารถนำมาประมวลได้ โดยมีคุณภาพที่ยอมรับได้



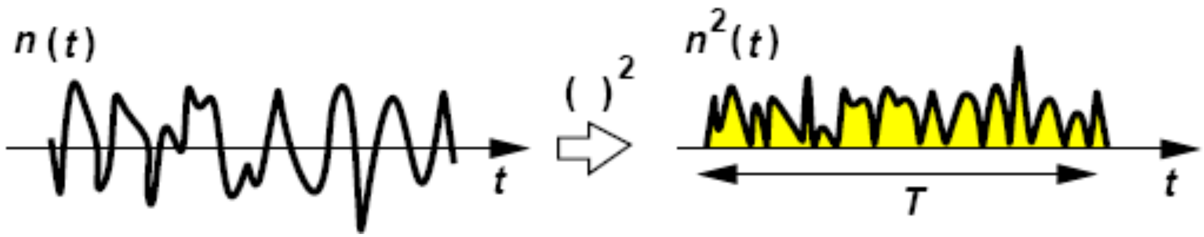
- เราสามารถรับสัญญาณที่มีกำลังงานน้อยกว่าระดับสัญญาณรบกวนได้หรือไม่ ?

Noise: Noise as a Random Process



Higher temperature

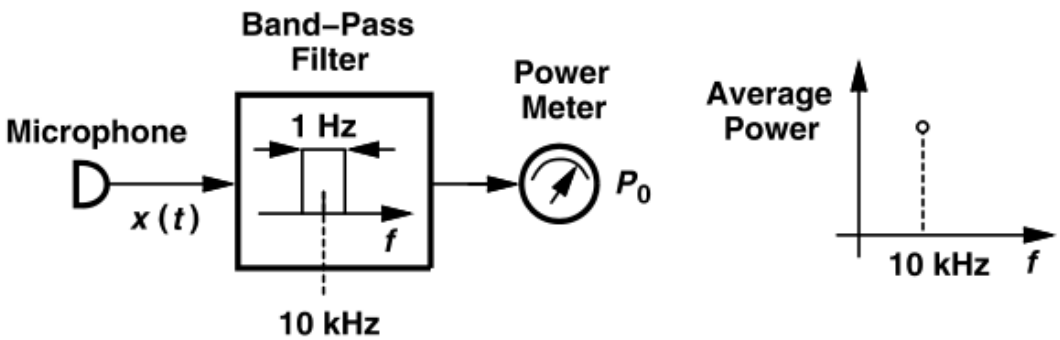
- The average current remains equal to V_B/R but the instantaneous current displays random values



$$P_n = \lim_{T\to\infty} \frac{1}{T}\int_0^T n^2(t)dt$$

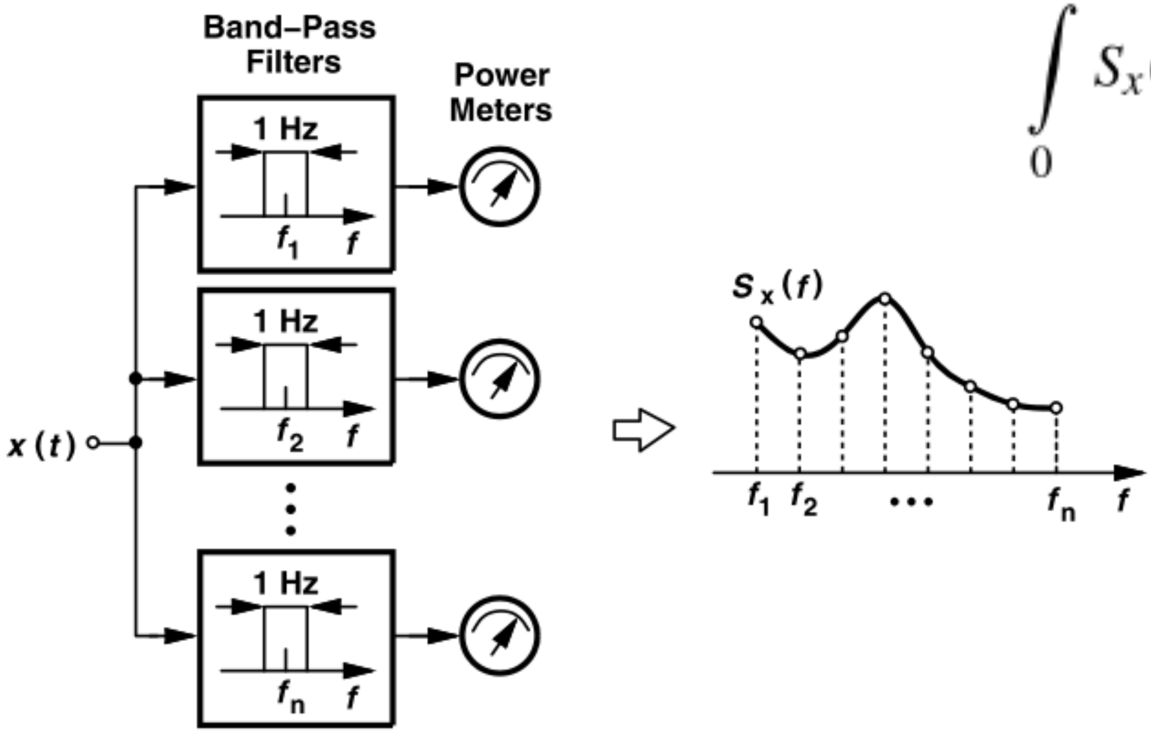
- *T* must be long enough to accommodate several cycles of the lowest frequency.

Noise spectrum or power spectral density (PSD)



(a)

$$\int_0^{\infty} S_x(f)df = \lim_{T\to\infty} \frac{1}{T} \int_0^{T} x^2(t)dt.$$



(b)

Effect of transfer function on noise

$$S_y(f) = S_x(f)|H(f)|^2$$

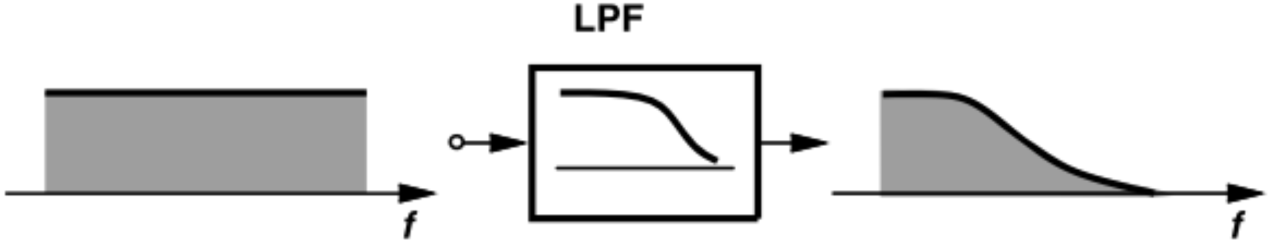


Figure 2.33 *Effect of low-pass filter on white noise.*

Noise in electronic devices

- Thermal noise of resistors
  - PSD

$$\overline{V_n^2} = 4kTR_1 \qquad \overline{I_n^2} = \overline{V_n^2}/R_1 = 4kT/R_1$$

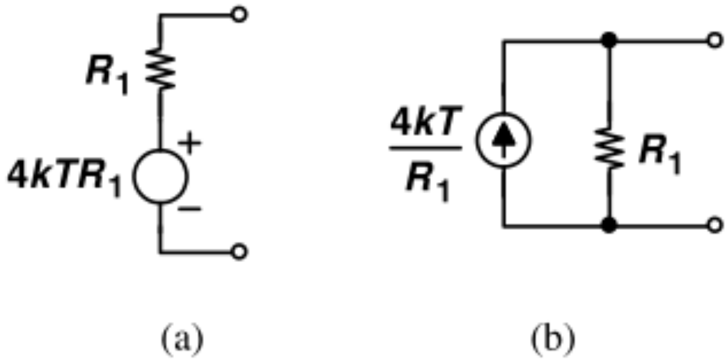


Figure 2.34 *(a) Thevenin and (b) Norton models of resistor thermal noise.*

Example 2.16

Sketch the PSD of the noise voltage measured across the parallel RLC tank depicted in Fig. 2.35(a).

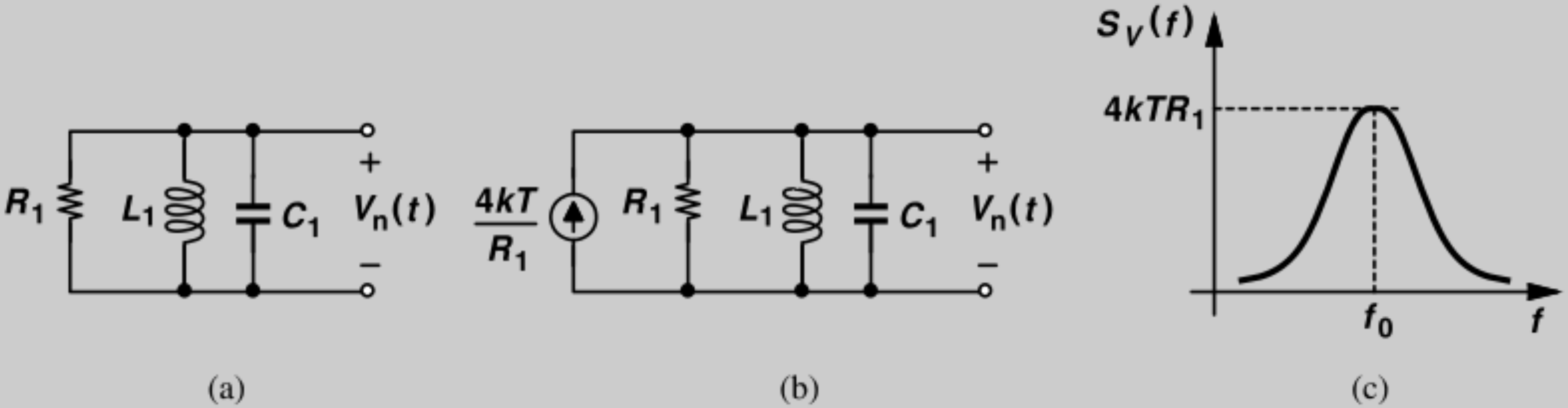


Figure 2.35 *(a) RLC tank, (b) inclusion of resistor noise, (c) output noise spectrum due to* R_1.

Solution:

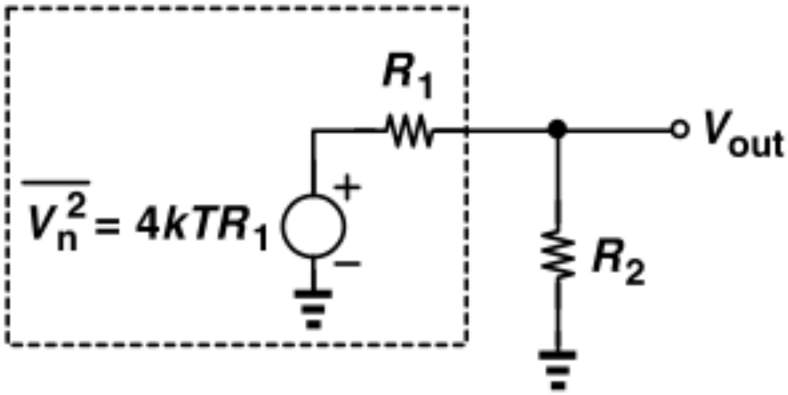
Modeling the noise of R_1 by a current source, $\overline{I_{n1}^2} = 4kT/R_1$, [Fig. 2.35(b)] and noting that the transfer function V_n/I_{n1} is, in fact, equal to the impedance of the tank, Z_T, we write from Eq. (2.86)

$$\overline{V_n^2} = \overline{I_{n1}^2}|Z_T|^2. \tag{2.87}$$

At $f_0 = (2\pi\sqrt{L_1C_1})^{-1}$, L_1 and C_1 resonate, reducing the circuit to only R_1. Thus, the output noise at f_0 is simply equal to $\overline{I_{n1}^2}R_1^2 = 4kTR_1$. At lower or higher frequencies, the impedance of the tank falls and so does the output noise [Fig. 2.35(c)].

Transfer of noise power

Suppose R_2 is held at T = 0 K



$$P_{R2} = \frac{\overline{V_{out}^2}}{R_2}$$

$$= \overline{V_n^2}\left(\frac{R_2}{R_1 + R_2}\right)^2 \frac{1}{R_2}$$

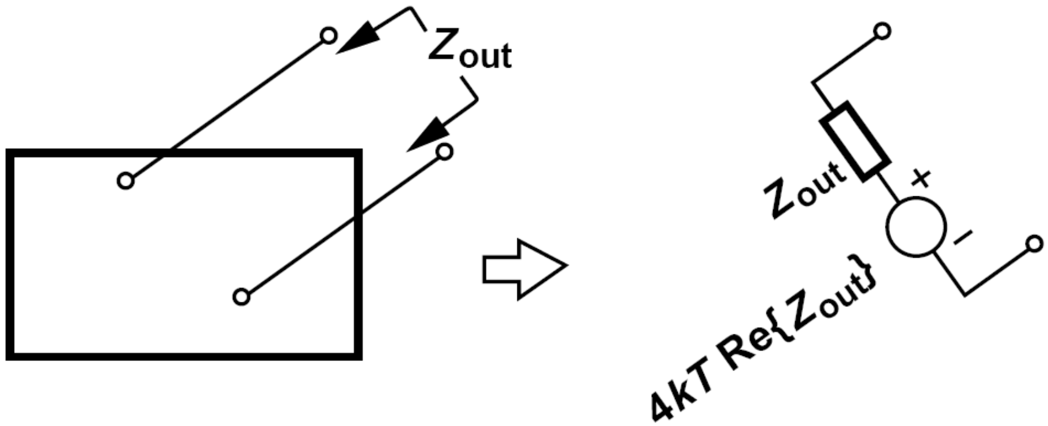
$$= 4kT\frac{R_1 R_2}{(R_1 + R_2)^2}.$$

This quantity reaches a maximum if $R_2 = R_1$:

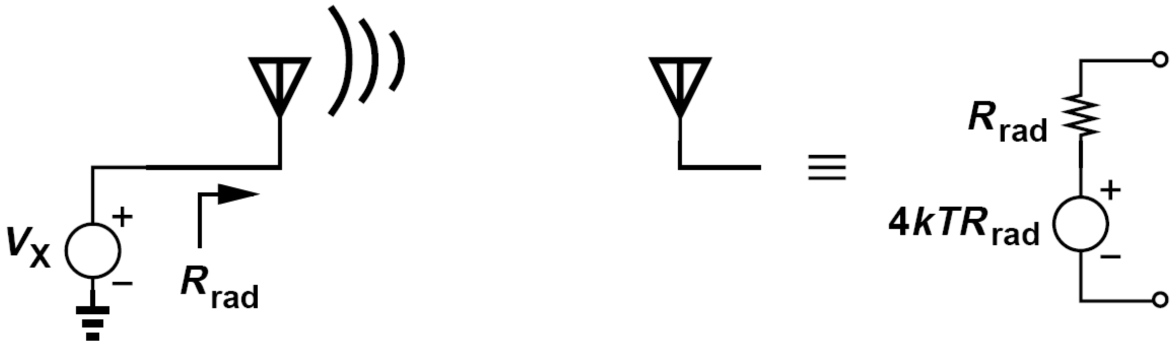
$$P_{R2,max} = kT.$$

Called the "available noise power," kT is independent of the resistor value and has the dimension of *power* per unit bandwidth. The reader can prove that $kT = -173.8$ dBm/Hz at $T = 300$ K.

Thermal noise in lossy circuits



- If the real part of the impedance seen between two terminals of a passive (reciprocal) network is equal to $Re\{Z_{out}\}$, then the PSD of the thermal noise seen between these terminals is given by $4kTRe\{Z_{out}\}$

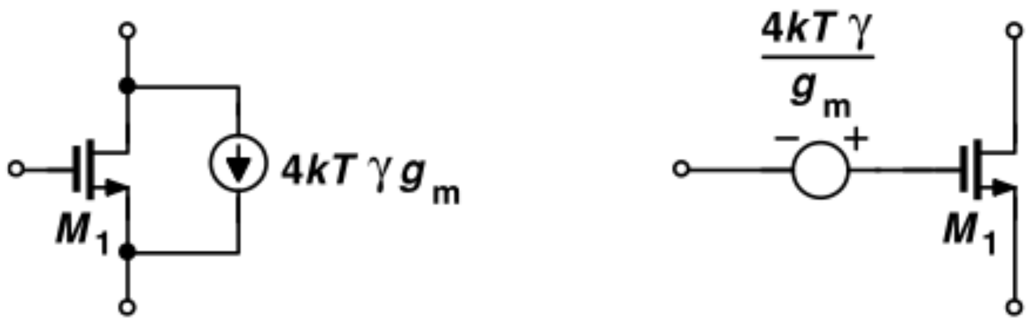


$$\overline{V_{n,ant}^2} = 4kTR_{rad}.$$

- An example of transmitting antenna, with radiation resistance R_{rad}

Thermal noise in MOSFETs

- Thermal noise of MOS transistors operating in the saturation region is approximated by a current source tied between the source and drain terminals, or can be modeled by a voltage source in series with gate.
- PSD $\overline{I_n^2} = 4kT\gamma g_m$ $\overline{V_n^2} = 4kT\gamma / g_m$



(a) (b)

where γ is the "excess noise coefficient" and g_m the transconductance.[17] The value of γ is 2/3 for long-channel transistors and may rise to even 2 in short-channel devices [4].

Thermal noise from gate resistance

Gate resistance

$$R_G = \frac{W}{L} R_{\square}$$

PSD

$$4kTR_G/3$$

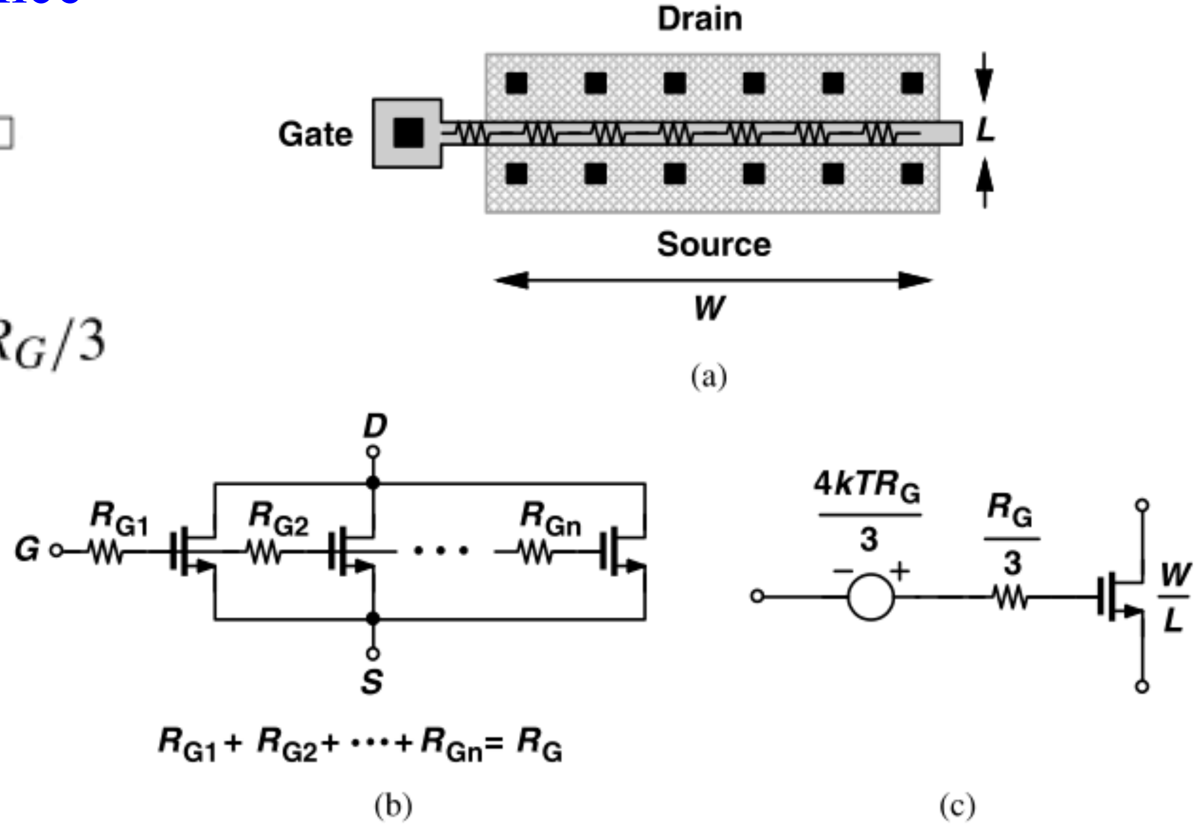


Figure 2.40 *(a) Gate resistance of a MOSFET, (b) equivalent circuit for noise calculation, (c) equivalent noise and resistance in lumped model.*

Flicker or 1/f noise in MOSFETs

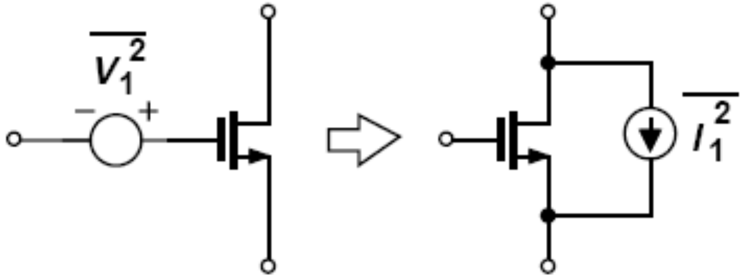
$$\overline{V_n^2} = \frac{K}{WLC_{ox}} \frac{1}{f}$$

where K is a process-dependent constant. In most CMOS technologies, K is lower for PMOS devices than for NMOS transistors because the former carry charge well below the silicon-oxide interface and hence suffer less from "surface states" (dangling bonds) [1]. The $1/f$ dependence means that noise components that vary slowly assume a large amplitude.

Can the flicker noise be modeled by a current source?

Yes, a MOSFET having a small-signal voltage source of magnitude V_1 in series with its gate is equivalent to a device with a current source of value $g_m V_1$ tied between drain and source. Thus,

$$\overline{I_1^2} = g_m^2 \frac{K}{WLC_{ox}} \frac{1}{f}$$



Sensitivity and dynamic range

- $$NF = \frac{SNR_{in}}{SNR_{out}} \qquad \text{SNR}_{in} = \frac{P_{sig}}{P_{RS} \cdot B}$$

$$\Rightarrow \quad \text{P}_{sig} = P_{RS} \cdot NF \cdot SNR_{out} \cdot B$$

P_{sig} is the Signal Power
P_{RS} is the source resistance Noise Power (per unit bandwidth)
B is the channel bandwidth

$$P_{sig}|_{dBm} = P_{RS}|_{dBm/Hz} + NF|_{dB} + SNR_{min}|_{dB} + 10log_{10}B$$

$$P_{RS} = kT = -174\ dBm/Hz \text{ at 300K}$$

$$\boxed{P_{in,min}|_{dBm} = -174\ dBm/Hz + NF|_{dB} + SNR_{min}|_{dB} + 10log_{10}B}$$

1/f noise corner frequency

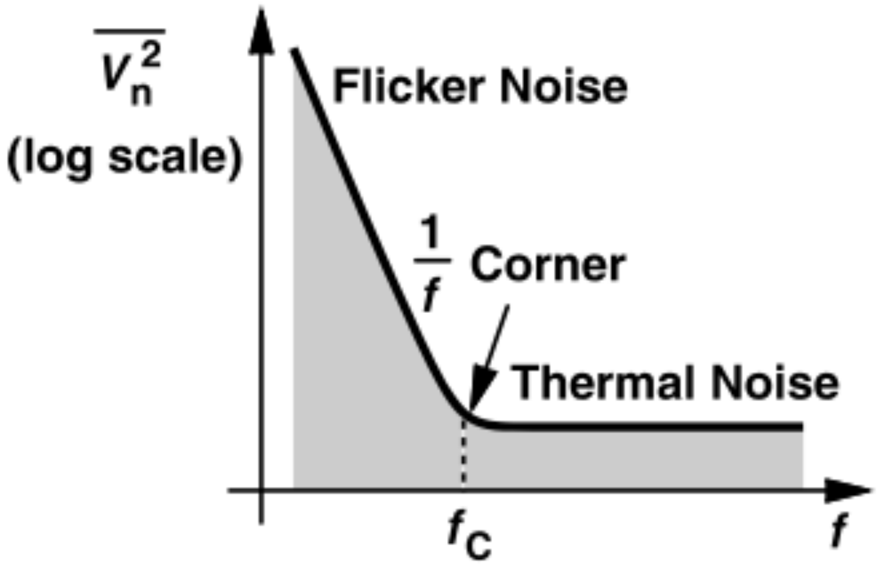


Figure 2.43 *Flicker noise corner frequency.*

$$\frac{K}{WLC_{ox}}\frac{1}{f_c}g_m^2 = 4kT\gamma g_m.$$

$$f_c = \frac{K}{WLC_{ox}}\frac{g_m}{4kT\gamma}.$$

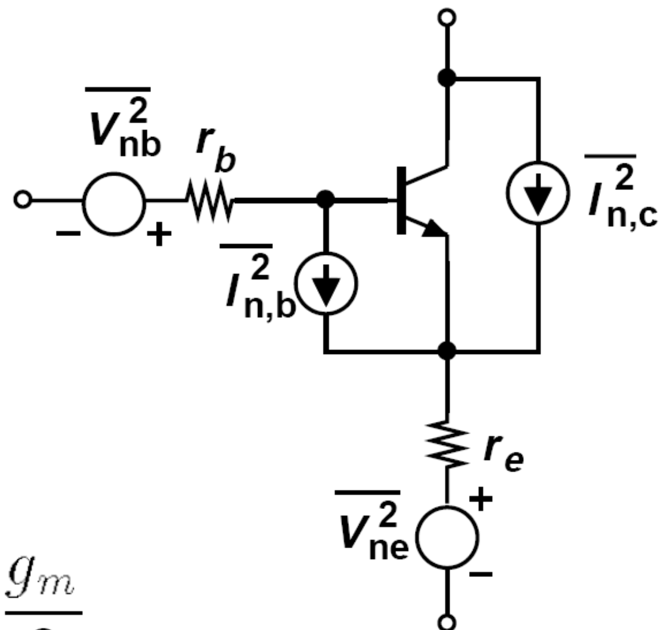
Noise in Bipolar Transistors

- Bipolar transistors contain physical resistances in their base, emitter, and collector regions, all of which generate thermal noise. Moreover, they also suffer from "shot noise" associated with the transport of carriers across the base-emitter junction.

$$\overline{I^2_{n,b}} = 2qI_B = 2q\frac{I_C}{\beta}$$

$$\overline{I^2_{n,c}} = 2qI_C,$$

$$g_m = I_C/(kT/q)$$



$$\overline{I^2_{n,c}} = 4kT\frac{g_m}{2}$$

- In low-noise circuits, the base resistance thermal noise and the collector current shot noise become dominant. For this reason, wide transistors biased at high current levels are employed.

Noise Figure

$$\mathrm{NF} = \frac{SNR_{in}}{SNR_{out}}$$

$$\mathrm{NF}|_{\mathrm{dB}} = 10 \log \frac{SNR_{in}}{SNR_{out}}.$$

- **Depends on not only the noise of the circuit under consideration but the SNR provided by the preceding stage**
- **If the input signal contains no noise, NF=∞**

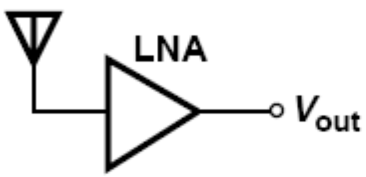
Calculation of Noise Figure

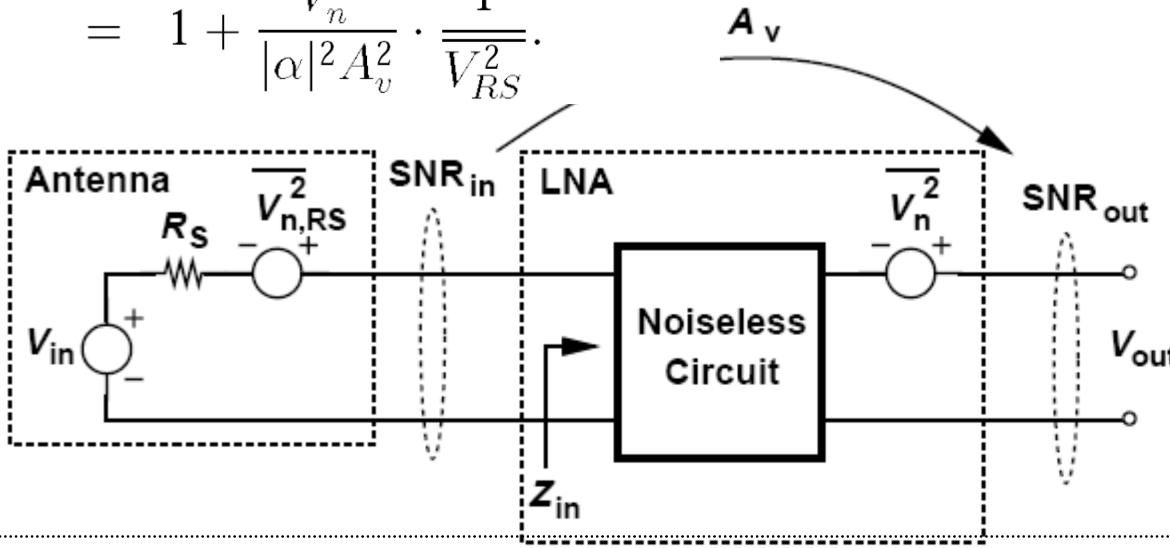
$$SNR_{in} = \frac{|\alpha|^2 V_{in}^2}{|\alpha|^2 \overline{V_{RS}^2}}$$

$$SNR_{out} = \frac{V_{in}^2 |\alpha|^2 A_v^2}{\overline{V_{RS}^2}|\alpha|^2 A_v^2 + \overline{V_n^2}}$$

$$\alpha = Z_{in}/(Z_{in} + R_S)$$

$$\begin{aligned} \mathrm{NF} &= \frac{V_{in}^2}{4kTR_S} \cdot \frac{\overline{V_{RS}^2}|\alpha|^2 A_v^2 + \overline{V_n^2}}{V_{in}^2 |\alpha|^2 A_v^2} \\ &= \frac{1}{\overline{V_{RS}^2}} \cdot \frac{\overline{V_{RS}^2}|\alpha|^2 A_v^2 + \overline{V_n^2}}{|\alpha|^2 A_v^2} \\ &= 1 + \frac{\overline{V_n^2}}{|\alpha|^2 A_v^2} \cdot \frac{1}{\overline{V_{RS}^2}}. \end{aligned}$$





- NF must be specified with respect to a source impedance-typically 50 Ω
- Reduce the right hand side to a simpler form:

$$\mathrm{NF} = \frac{1}{4kTR_S} \cdot \frac{\overline{V_{n,out}^2}}{A_0^2}$$

Calculation of NF: Summary

Calculation of NF

- **Divide total output noise by the gain from V_{in} to V_{out} and normalize the result to the noise of R_s**

- **Calculate the output noise due to the amplifier, divide it by the gain, normalize it to $4kTR_s$ and add 1 to the result**

- **Valid even if no actual power is transferred. So long as the derivations incorporate noise and signal voltages, no inconsistency arises in the presence of impedance mismatches or even infinite input impedances.**

Example 2.20

Compute the noise figure of a shunt resistor R_P with respect to a source impedance R_S [Fig. 2.49(a)].

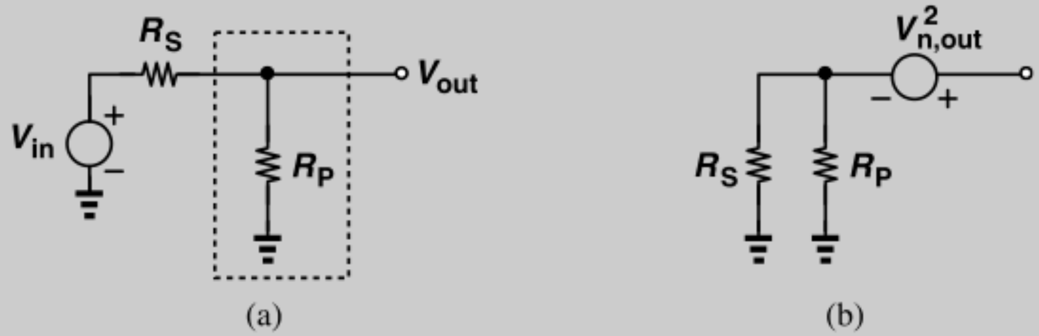


Figure 2.49 *(a) Circuit consisting of a single parallel resistor, (b) model for NF calculation.*

From Fig. 2.49(b), the total output noise voltage is obtained by setting V_{in} to zero:

$$\overline{V^2_{n,out}} = 4kT(R_S||R_P).$$

The gain is equal to

$$A_0 = \frac{R_P}{R_P + R_S}.$$

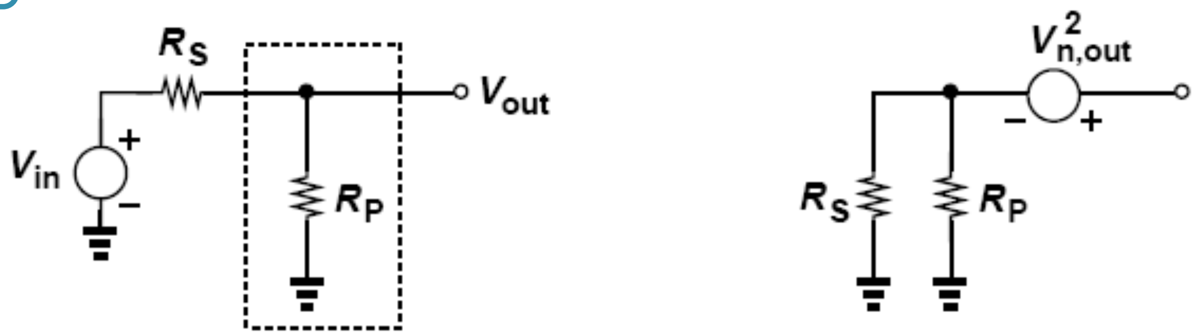
Thus,

$$\begin{aligned} \mathrm{NF} &= 4kT(R_S||R_P)\frac{(R_S + R_P)^2}{R_P^2}\frac{1}{4kTR_S} \\ &= 1 + \frac{R_S}{R_P}. \end{aligned}$$

Example of Noise Figure Calculation

Compute the noise figure of a shunt resistor R_P with respect to a source impedance R_S

Solution:



Setting V_{in} to zero:

$$\overline{V_{n,out}^2} = 4kT(R_S||R_P) \qquad A_0 = \frac{R_P}{R_P + R_S}$$

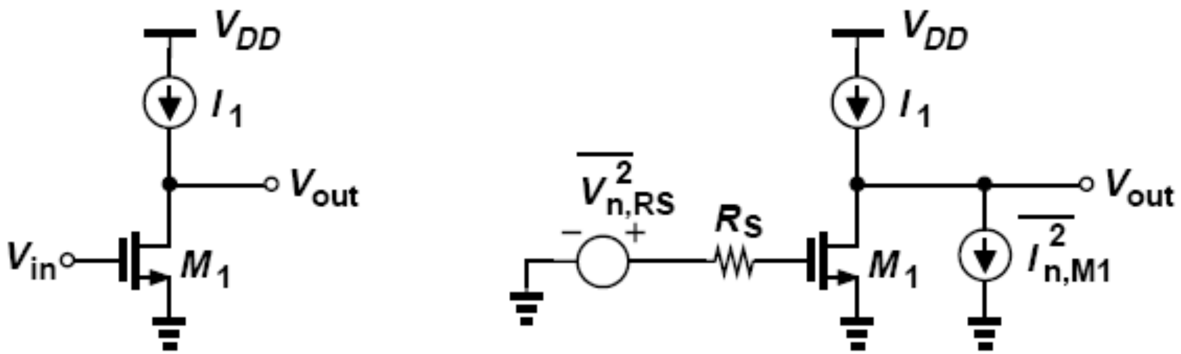
$$\begin{aligned} \mathrm{NF} &= 4kT(R_S||R_P)\frac{(R_S+R_P)^2}{R_P^2}\frac{1}{4kTR_S} \\ &= 1 + \frac{R_S}{R_P}. \end{aligned}$$

- NF is minimized by maximizing Rp
- For max. power transfer => Rp=Rs => NF = 2 or 3 dB

Example of Noise Figure Calculation

Determine the noise figure of the common-source stage shown in below (left) with respect to a source impedance R_S. Neglect the capacitances and flicker noise of M_1 and assume I_1 is ideal.

Solution:

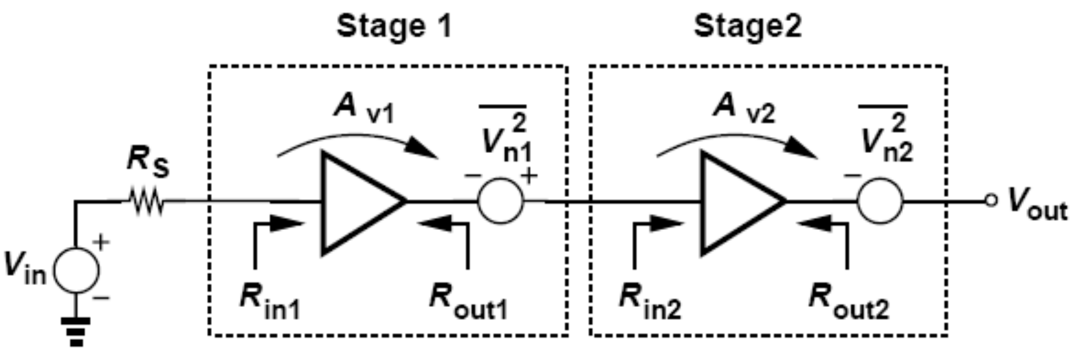


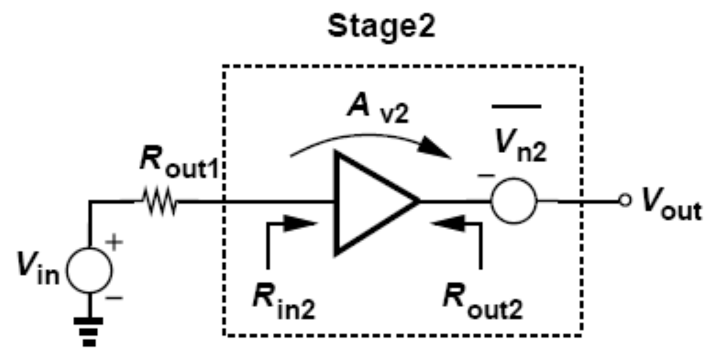
$$\mathrm{NF} = \frac{4kT\gamma g_m r_O^2 + 4kTR_S(g_m r_O)^2}{(g_m r_O)^2} \cdot \frac{1}{4kTR_S}$$

$$= \frac{\gamma}{g_m R_S} + 1.$$

This result implies that the NF falls as R_S rises. Does this mean that, even though the amplifier remains unchanged, the overall system noise performance improves as R_S increases?!

Noise Figure of Cascaded Stages (Ⅰ)





$$A_0 = \frac{V_{out}}{V_{in}} = \frac{R_{in1}}{R_{in1}+R_S} A_{v1} \frac{R_{in2}}{R_{in2}+R_{out1}} A_{v2}$$

$$\overline{V_{n,out}^2} = \overline{V_{n2}^2} + \overline{V_{n1}^2} \frac{R_{in2}^2}{(R_{in2}+R_{out1})^2} A_{v2}^2$$

$$\begin{aligned}
\mathrm{NF_{tot}} &= 1 + \frac{\overline{V_{n,out}^2}}{A_0^2} \cdot \frac{1}{4kTR_S} \\
&= 1 + \frac{\overline{V_{n1}^2}}{\left(\frac{R_{in1}}{R_{in1}+R_S}\right)^2 A_{v1}^2} \cdot \frac{1}{4kTR_S} \\
&+ \frac{\overline{V_{n2}^2}}{\left(\frac{R_{in1}}{R_{in1}+R_S}\right)^2 A_{v1}^2 \left(\frac{R_{in2}}{R_{in2}+R_{out1}}\right)^2 A_{v2}^2} \cdot \frac{1}{4kTR_S}
\end{aligned}$$

Noise Figure of Cascaded Stages (Ⅱ)

$$NF_2 = 1 + \frac{\overline{V_{n2}^2}}{\frac{R_{in2}^2}{(R_{in2}+R_{out1})^2}A_{v2}^2}\frac{1}{4kTR_{out1}} \quad NF_{tot} = NF_1 + \frac{NF_2 - 1}{\frac{R_{in1}^2}{(R_{in1}+R_S)^2}A_{v1}^2\frac{R_S}{R_{out1}}}$$

$$P_{out,av} = V_{in}^2\frac{R_{in1}^2}{(R_S+R_{in1})^2}A_{v1}^2 \cdot \frac{1}{4R_{out1}}.$$

$$P_{S,av} = \frac{V_{in}^2}{4R_S}$$

This quantity is in fact the "available power gain" of the first stage, defined as the "available power" at its output, $P_{out,av}$ (the power that it would deliver to a matched load) divided by the available source power, $P_{S,av}$ (the power that the source would deliver to a matched load).

$$NF_{tot} = NF_1 + \frac{NF_2 - 1}{A_{P1}}$$

$$NF_{tot} = 1 + (NF_1 - 1) + \frac{NF_2 - 1}{A_{P1}} + \cdots + \frac{NF_m - 1}{A_{P1}\cdots A_{P(m-1)}}.$$

Called "Friis' equation", this result suggests that the noise contributed by each stage decreases as the total gain preceding that stage increases, implying that the first few stages in a cascade are the most critical.

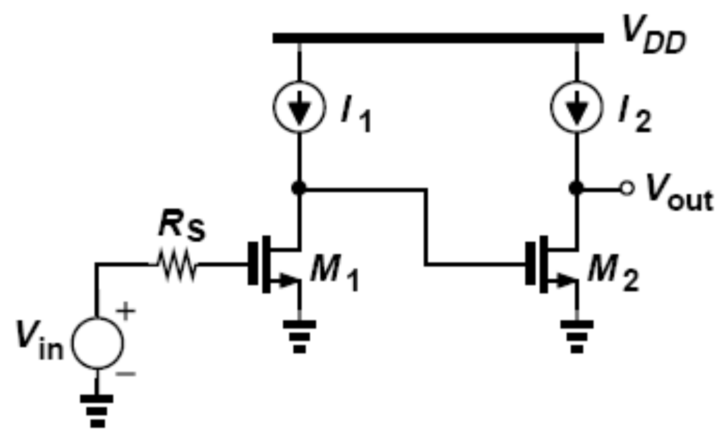
Example of Noise Figure of Cascaded Stages

Determine the *NF* of the cascade of common-source stages shown in figure below. Neglect the transistor capacitances and flicker noise.

Solution:

$$R_{in1} = R_{in2} = \infty$$

$$NF = 1 + \frac{\overline{V_{n1}^2}}{A_{v1}^2}\frac{1}{4kTR_S} + \frac{\overline{V_{n2}^2}}{A_{v1}^2 A_{v2}^2}\frac{1}{4kTR_S}$$

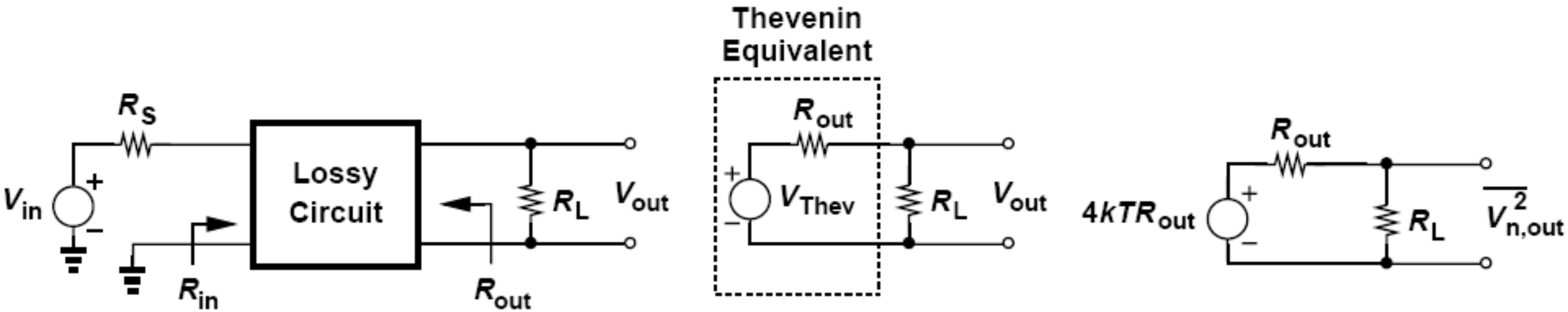


where

$$\overline{V_{n1}^2} = 4kT\gamma g_{m1} r_{O1}^2,\ \overline{V_{n2}^2} = 4kT\gamma g_{m2} r_{O2}^2,\ A_{v1} = g_{m1} r_{O1},\ \text{and}\ A_{v2} = g_{m2} r_{O2}$$

$$NF = 1 + \frac{\gamma}{g_{m1} R_S} + \frac{\gamma}{g_{m1}^2 r_{O1}^2 g_{m2} R_S}$$

Noise Figure of Lossy Circuits



The power loss is calculated as:

$$L = P_{in}/P_{out}$$

$$L = \frac{V_{in}^2}{V_{Thev}^2}\frac{R_{out}}{R_S}$$

$$\overline{V_{n,out}^2} = 4kTR_{out}\frac{R_L^2}{(R_L + R_{out})^2}$$

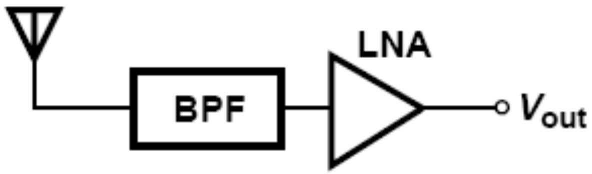
$$A_0 = \frac{V_{Thev}}{V_{in}}\frac{R_L}{R_L + R_{out}}$$

$$\begin{aligned} NF &= 4kTR_{out}\frac{V_{in}^2}{V_{Thev}^2}\frac{1}{4kTR_S} \\ &= L. \end{aligned}$$

Example of Noise Figure of Lossy Circuits

The receiver shown below incorporates a front-end band-pass filter (BPF) to suppress some of the interferers that may desensitize the LNA. If the filter has a loss of *L* and the LNA a noise figure of NF_{LNA}, calculate the overall noise figure.

Solution:

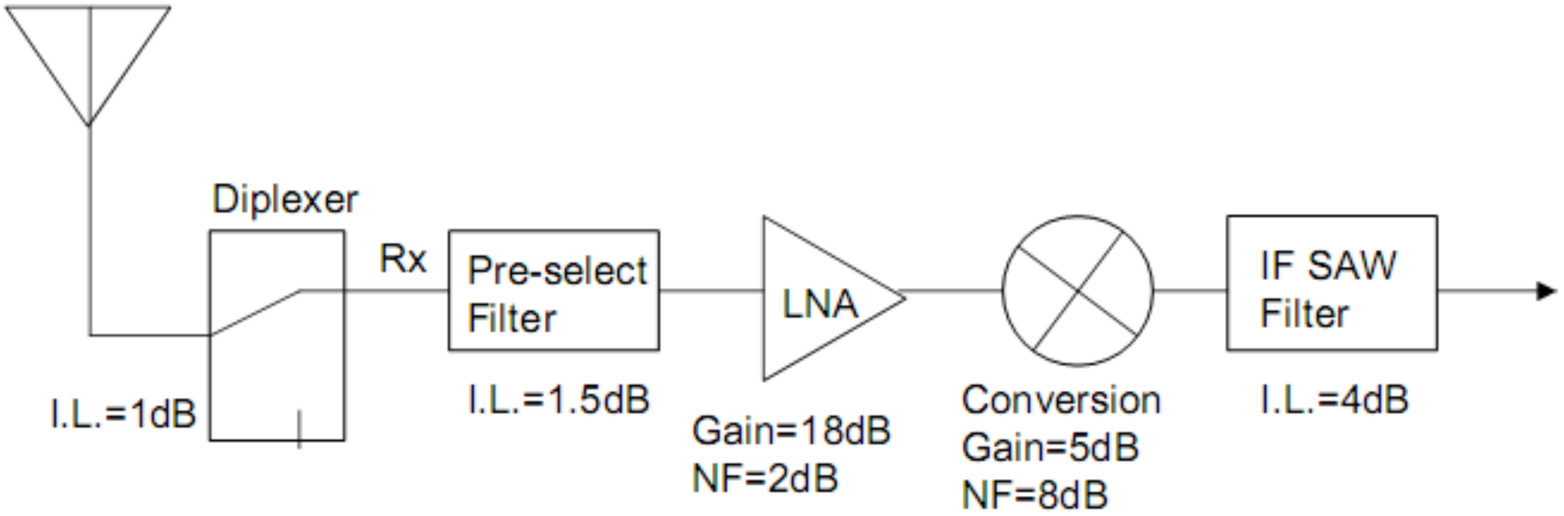


Denoting the noise figure of the filter by NF_{filt}, we write Friis' equation as

$$\begin{aligned} NF_{tot} &= NF_{filt} + \frac{NF_{LNA} - 1}{L^{-1}} \\ &= L + (NF_{LNA} - 1)L \\ &= L \cdot NF_{LNA}, \end{aligned}$$

where NF_{LNA} is calculated with respect to the output resistance of the filter. For example, if L = 1.5 dB and NF_{LNA} = 2 dB, then NF_{tot} = 3.5 dB.

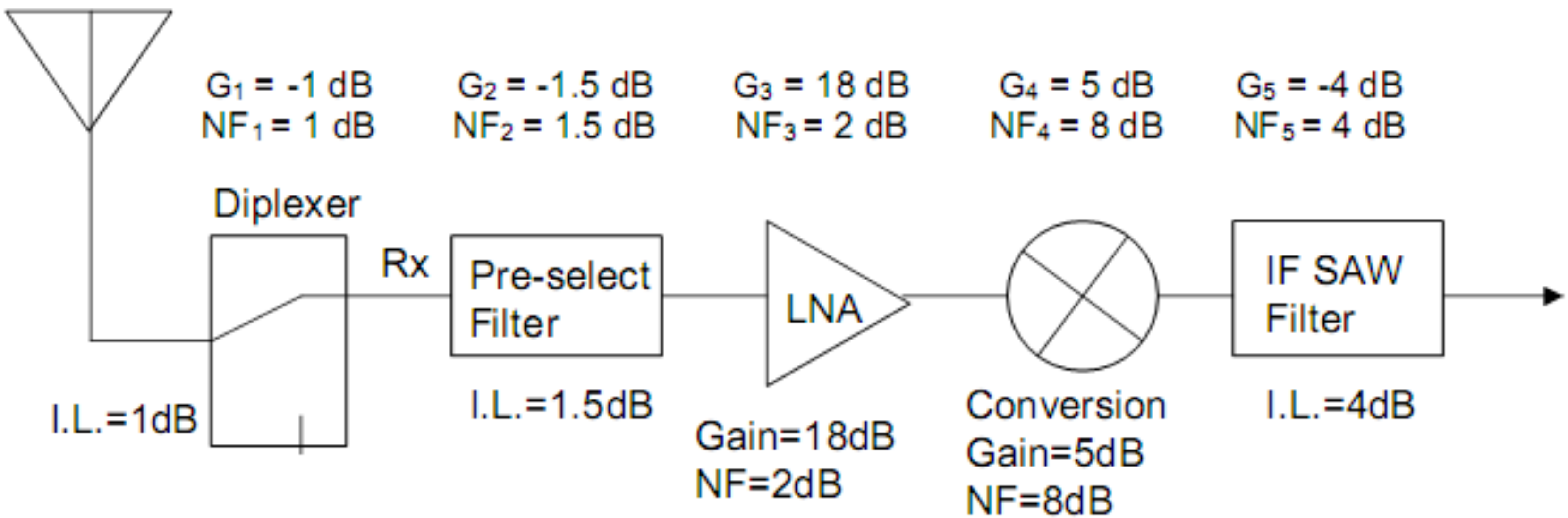
Example: NF of a receiver chain



Overall Noise Figure = ?

Example: NF of a receiver chain

Method I: Brute force Frii's Formula



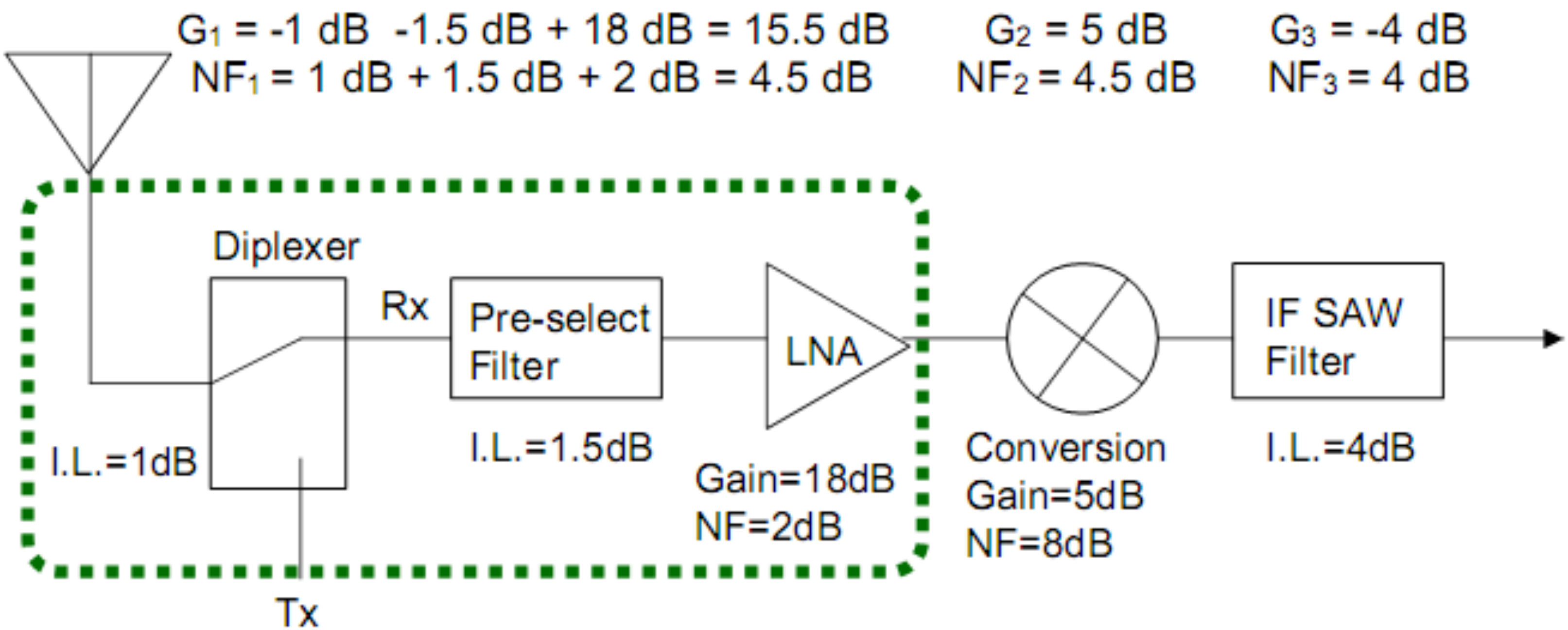
$$F_{tot} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 \cdot G_2} + \frac{F_4 - 1}{G_1 \cdot G_2 \cdot G_3} + \frac{F_5 - 1}{G_1 \cdot G_2 \cdot G_3 \cdot G_4}$$

$$F_{tot} = 10^{\frac{1}{10}} + \frac{10^{\frac{1.5}{10}} - 1}{10^{\frac{-1}{10}}} + \frac{10^{\frac{2}{10}} - 1}{10^{\frac{-1}{10}} \cdot 10^{\frac{-1.5}{10}}} + \frac{10^{\frac{8}{10}} - 1}{10^{\frac{-1}{10}} \cdot 10^{\frac{-1.5}{10}} \cdot 10^{\frac{18}{10}}} + \frac{10^{\frac{4}{10}} - 1}{10^{\frac{-1}{10}} \cdot 10^{\frac{-1.5}{10}} \cdot 10^{\frac{18}{10}} \cdot 10^{\frac{5}{10}}}$$

$$\boxed{F_{tot} = 2.98 => NF_{tot} = 10 log F_{tot} = 4.7\ dB}$$

Example: NF of a receiver chain

Method II: Combine Lossy stage and active stages, then Frii's formula



$$F_{tot} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_2} = 10^{\frac{4.5}{10}} + \frac{10^{\frac{8}{10}} - 1}{10^{\frac{15.5}{10}}} + \frac{10^{\frac{4}{10}} - 1}{10^{\frac{15.5}{10}} \times 10^{\frac{5}{10}}} = 2.968$$

$$\boxed{NF_{tot} = 10log(2.968) = 4.725\ dB}$$

Sensitivity and Dynamic Range: Sensitivity

- **The sensitivity is defined as the minimum signal level that a receiver can detect with "acceptable quality."**

$$NF = \frac{SNR_{in}}{SNR_{out}} = \frac{P_{sig}/P_{RS}}{SNR_{out}}$$

$$P_{sig} = P_{RS} \cdot NF \cdot SNR_{out}$$

$$P_{sig,tot} = P_{RS} \cdot NF \cdot SNR_{out} \cdot B$$

$$P_{sen}|_{dBm} = P_{RS}|_{dBm/Hz} + NF|_{dB} + SNR_{min}|_{dB} + 10\log B$$

$$P_{sen} = \underbrace{-174\ \text{dBm/Hz} + NF + 10\log B}_{\text{Noise Floor}} + SNR_{min}$$

Noise Floor

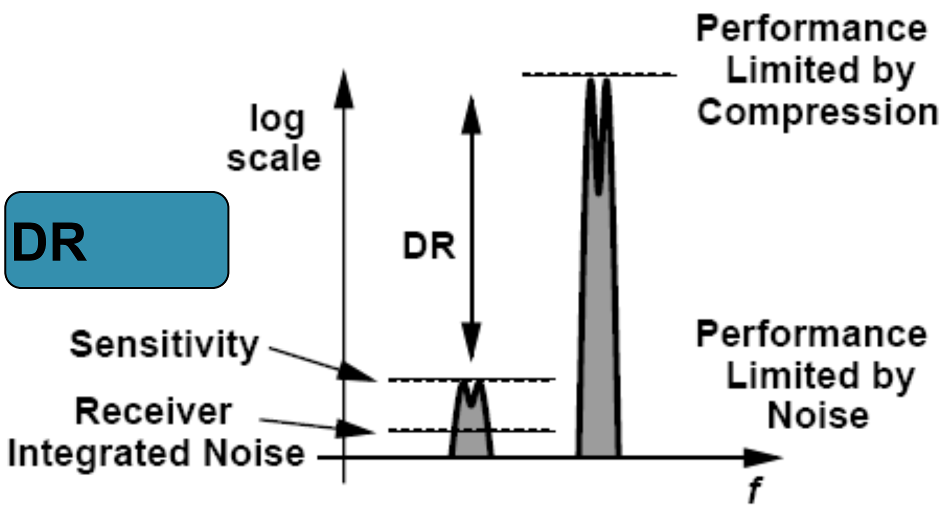
Example of Sensitivity

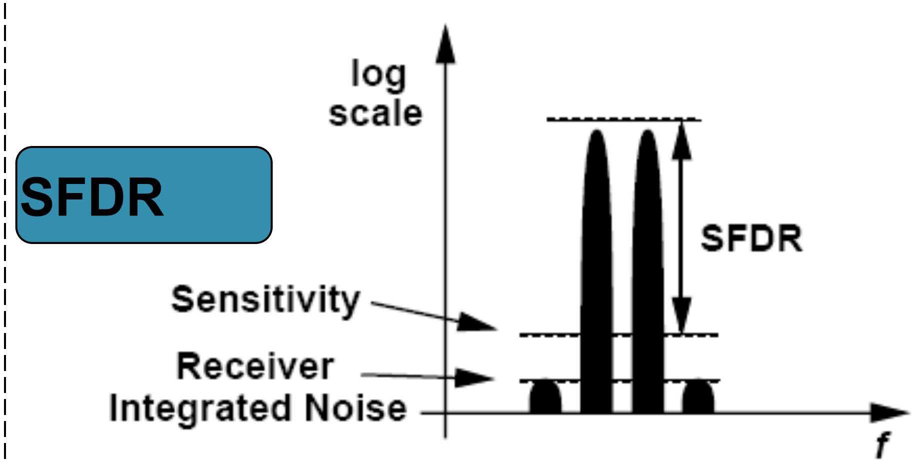
A GSM receiver requires a minimum *SNR* of 12 dB and has a channel bandwidth of 200 kHz. A wireless LAN receiver, on the other hand, specifies a minimum *SNR* of 23 dB and has a channel bandwidth of 20 MHz. Compare the sensitivities of these two systems if both have an *NF* of 7 dB.

Solution:

For the GSM receiver, P_{sen} = -102 dBm, whereas for the wireless LAN system, P_{sen} = -71 dBm. Does this mean that the latter is inferior? No, the latter employs a much wider bandwidth and a more efficient modulation to accommodate a data rate of 54 Mb/s. The GSM system handles a data rate of only 270 kb/s. In other words, specifying the sensitivity of a receiver without the data rate is not meaningful.

Dynamic Range vs. SFDR





- **Dynamic Range:**

$$DR = \frac{Max\ Tolerable\ Signal}{Min\ Detectable\ Signal}$$

- **SFDR:**

  Lower end equal to sensitivity.

  Higher end defined as maximum input level in a *two-tone* test for which the third-order IM products do not exceed the integrated noise of the receiver

SFDR Calculation

Refer output IM magnitudes to input:

$$P_{IIP3} = P_{in} + \frac{P_{out} - P_{IM,out}}{2}$$

$$P_{IM,in} = P_{IM,out} - G. \qquad P_{in} = P_{out} - G.$$

$$P_{IIP3} = P_{in} + \frac{P_{in} - P_{IM,in}}{2}$$

$$= \frac{3P_{in} - P_{IM,in}}{2},$$

$$P_{in} = \frac{2P_{IIP3} + P_{IM,in}}{3}.$$

$$P_{in,max} = \frac{2P_{IIP3} + (-174\,\text{dBm} + NF + 10\log B)}{3}.$$

$$SFDR = P_{in,max} - (-174\,\text{dBm} + NF + 10\log B + SNR_{min})$$

$$= \frac{2(P_{IIP3} + 174\,\text{dBm} - NF - 10\log B)}{3} - SNR_{min}.$$

Example Comparing SFDR and DR

The upper end of the dynamic range is limited by intermodulation in the presence of two interferers or desensitization in the presence of one interferer. Compare these two cases and determine which one is more restrictive.

Solution:

$$P_{1-dB} \overset{?}{\gtrless} P_{in,max}$$

Since $P_{1-dB} = P_{IIP3} - 9.6 \text{ dB},$

$$P_{IIP3} - 9.6 \text{ dB} \overset{?}{\gtrless} \frac{2P_{IIP3} + (-174 \text{ dBm} + NF + 10 \log B)}{3}$$

$$P_{IIP3} - 28.8 \text{ dB} \overset{?}{\gtrless} -174 \text{ dBm} + NF + 10 \log B$$

Noise floor

$$P_{1-dB} > P_{in,max}$$

- **SFDR is a more stringent characteristic of system than DR**

Blocking Dynamic range

Blocking Dynamic Range (BDR)

Here we assume, $P_{in,max} = P_{1dB}$,

Hence,

$$BDR = P_{1dB} - F - SNR_{min}$$

(NB: theoretically, $P_{1dB} \sim P_{IIP3}$ - 10 dB)

+ Example: Dynamic range

Consider an amplifier with the following specification

Gain	BW	NF	P_{1dB}	IIP3
30 dB	200 MHz	6 dB	30 dBm	40 dBm

MDS is 6 dB above *thermal noise power level*

Blocking Dynamic Range = ?
Spurious Free Dynamic Range = ?

Example: Dynamic range

$SNR_{min} = 6\ dB$

Noise power level:
$F = -174\ dBm/Hz + NF + 10logB = -174 + 6 + 10log(2 \times 10^8) = -85\ dBm$

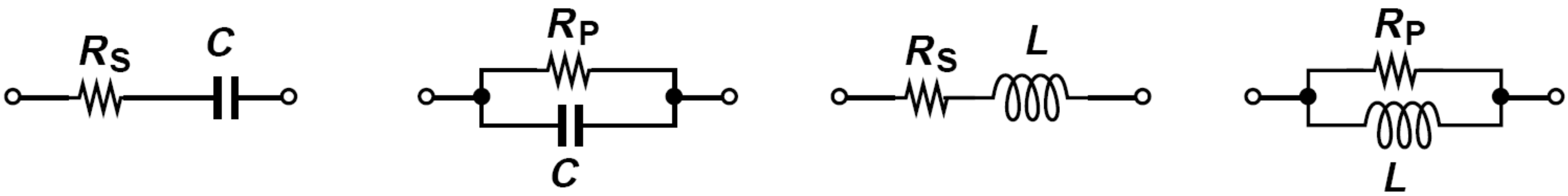
Minimum Detectable Signal: $P_{in,min} = F + SNR_{min} = -85 + 6 = -79\ dBm$

Blocking Dynamic Range: $BDR = P_{1dB} - P_{in,min} = 30 - (-79) = 109\ dB$

$$SFDR = \frac{2(P_{IIP3} - F)}{3} - SNR_{min} = \frac{2(40 - (-85))}{3} - 6 = 77.3\ dB$$

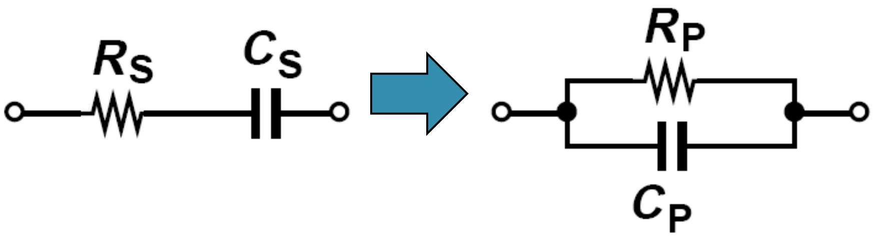
Passive Impedance Transformation: Quality Factor

- **Quality Factor, *Q*, indicates how close to ideal an energy-storing device is.**



$$Q_S = \frac{\frac{1}{C\omega}}{R_S} \qquad Q_P = \frac{R_P}{\frac{1}{C\omega}} \qquad Q_S = \frac{L\omega}{R_S} \qquad Q_P = \frac{R_P}{L\omega}$$

Series-to-Parallel Conversion



$$\frac{R_S C_S s + 1}{C_S s} = \frac{R_P}{R_P C_P s + 1}$$

$$R_P C_S j\omega = 1 - R_P C_P R_S C_S \omega^2 + (R_P C_P + R_S C_S) j\omega.$$

$$\begin{aligned} R_P C_P R_S C_S \omega^2 &= 1 \\ R_P C_P + R_S C_S - R_P C_S &= 0. \end{aligned}$$

$Q_s = Q_p$

$$R_P = \frac{1}{R_S C_S^2 \omega^2} + R_S$$

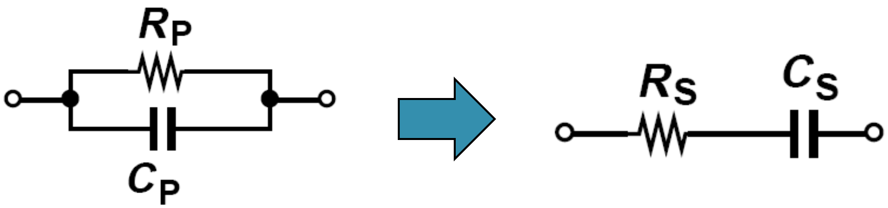
$$R_P = (Q_S^2 + 1) R_S$$

$$C_P = \frac{Q_S^2}{Q_S^2 + 1} C_S$$

$$Q_S^2 \gg 1$$

$$\begin{aligned} R_P &\approx Q_S^2 R_S \\ C_P &\approx C_S. \end{aligned}$$

Parallel-to-Series Conversion



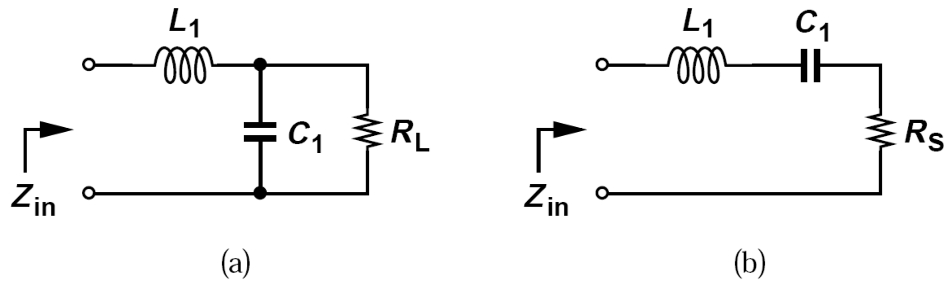
$$R_S = \frac{R_P}{Q_P^2}$$

$$C_s = C_P$$

- Series-to-Parallel Conversion: will retain the value of the capacitor but raises the resistance by a factor of Q_s^2
- Parallel-to-Series Conversion: will reduce the resistance by a factor of Q_P^2

Basic Matching Networks

$$Z_{in}(j\omega) = \frac{R_L(1 - L_1C_1\omega^2) + jL_1\omega}{1 + jR_LC_1\omega}$$



Thus,

$$Re\{Z_{in}\} = \frac{R_L}{1 + R_L^2C_1^2\omega^2}$$

$$= \frac{R_L}{1 + Q_P^2},$$

R_L transformed down by a factor

$$L_1 = \frac{R_L^2C_1}{1 + R_L^2C_1^2\omega^2}$$

$$= \frac{R_L^2C_1}{1 + Q_P^2}.$$

Setting imaginary part to zero

If $Q_P^2 \gg 1$

$$Re\{Z_{in}\} \approx \frac{1}{R_LC_1^2\omega^2}$$

$$L_1 = \frac{1}{C_1\omega^2}.$$

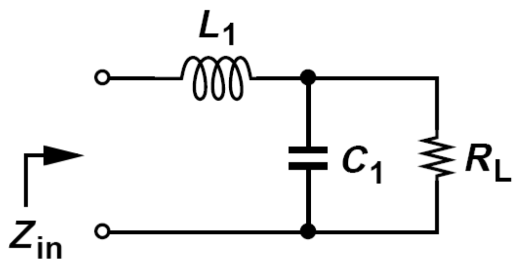
Example of Basic Matching Networks

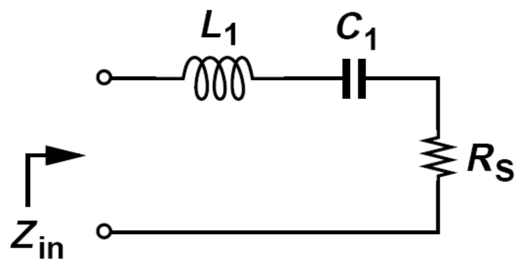
Design the matching network of figure above so as to transform R_L = 50 Ω to 25 Ω at a center frequency of 5 GHz.

Solution:

$$Re\{Z_{in}\} \approx \frac{1}{R_L C_1^2 \omega^2}$$

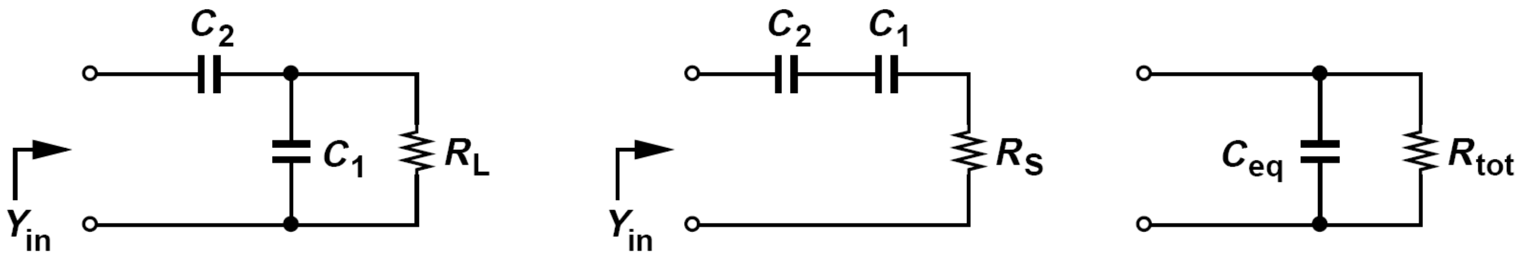
$$L_1 = \frac{1}{C_1 \omega^2}.$$





Assuming Q_P^2 >> 1, we have C_1 = 0:90 pF and L_1 = 1.13 nH, respectively. Unfortunately, however, Q_P = 1.41, indicating the Q_P^2 >> 1 approximation cannot be used. We thus obtain C_1 = 0:637 pF and L_1 = 0:796 nH.

Transfer a Resistance to a Higher Value



If $Q^2 \gg 1$ $\qquad R_S \approx [R_L(C_1\omega)^2]^{-1}$ $\qquad C_S \approx C_1$

Viewing C_2 and C_1 as one capacitor, C_{eq}

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

$$\begin{aligned} R_{tot} &= \frac{1}{R_S(C_{eq}\omega)^2} \\ &= \left(1 + \frac{C_1}{C_2}\right)^2 R_L \end{aligned}$$

RL boosted

For low *Q* values

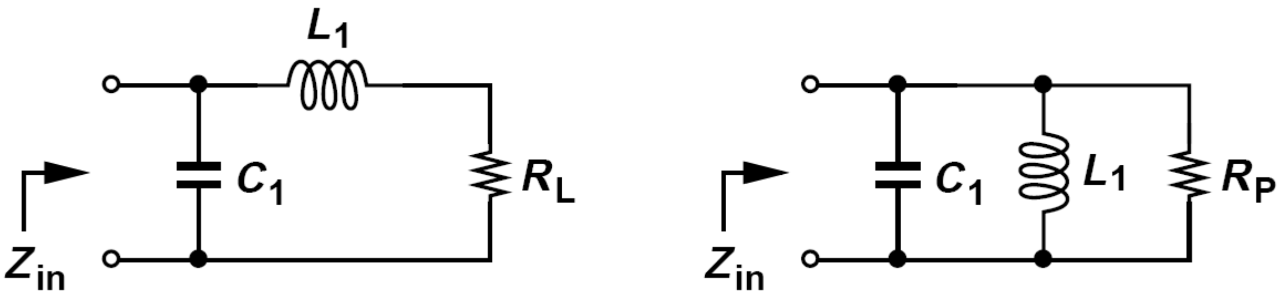
$$Y_{in} = \frac{j\omega C_2(1 + j\omega R_L C_1)}{1 + R_L(C_1 + C_2)j\omega}$$

$$\begin{aligned} R_{tot} &= \frac{1}{Re\{Y_{in}\}} \\ &= \frac{1}{R_L C_2^2 \omega^2} + R_L\left(1 + \frac{C_1}{C_2}\right)^2 \end{aligned}$$

Another Example of Basic Matching Networks

Determine how the circuit shown below transforms R_L.

Solution:

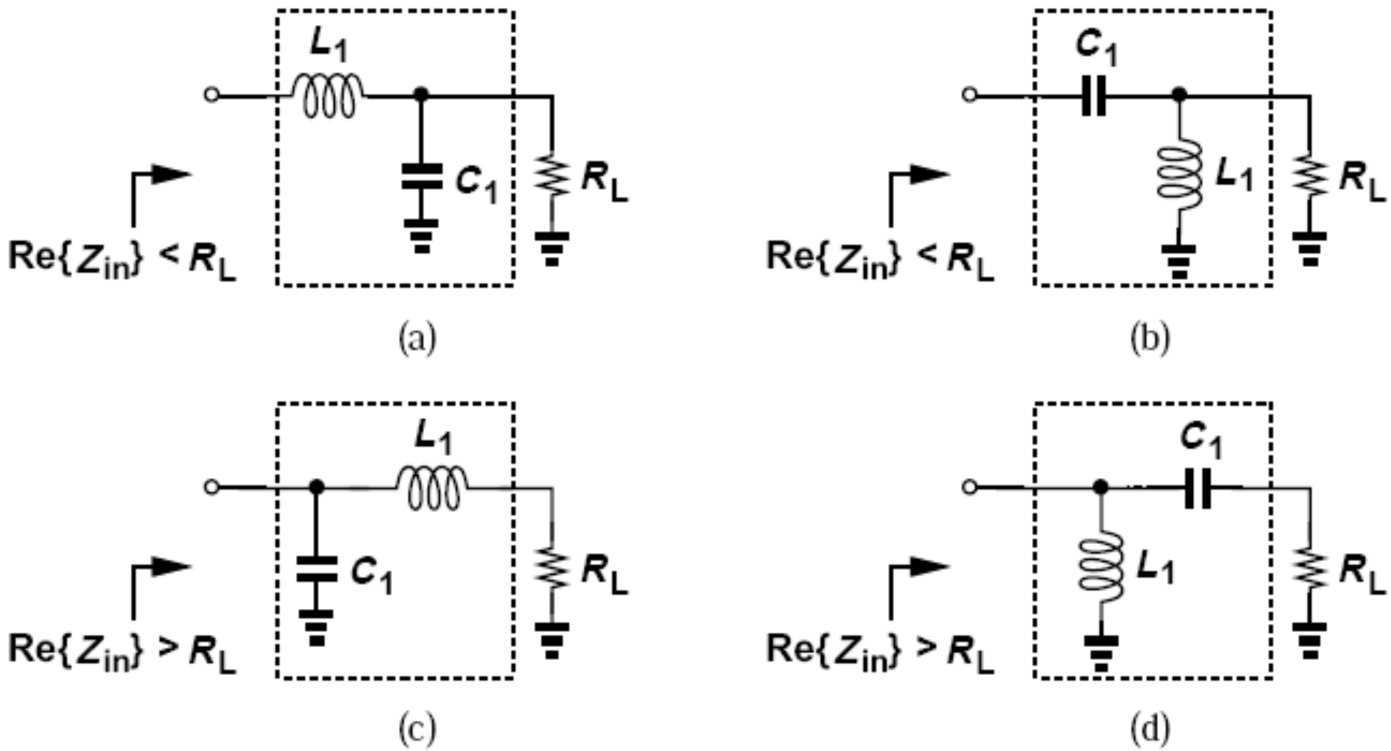


We postulate that conversion of the L_1-R_L branch to a parallel section produces a higher resistance. If $Q_S^2 = (L_1\omega/R_L)^2 >> 1$, then the equivalent parallel resistance is

$$
\begin{aligned}
R_P &= Q_S^2 R_L \\
&= \frac{L_1^2\omega^2}{R_L}.
\end{aligned}
$$

The parallel equivalent inductance is approximately equal to L_1 and is cancelled by C_1

L-Sections



For example, in (a), we have:

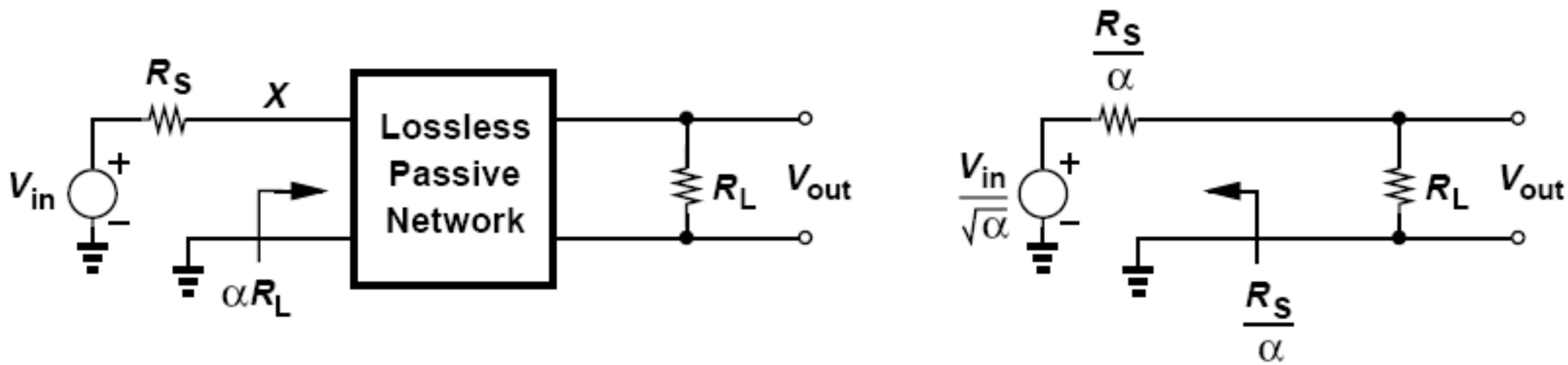
$$\frac{V_{out}}{V_{in}} = \sqrt{\frac{R_L}{Re\{Z_{in}\}}}. \qquad \frac{I_{out}}{I_{in}} = \sqrt{\frac{Re\{Z_{in}\}}{R_L}}$$

a network transforming R_L to a lower value “amplifies” the voltage and attenuates the current by the above factor.

Example of L-Sections

A closer look at the L-sections (a) and (c) suggests that one can be obtained from the other by swapping the input and output ports. Is it possible to generalize this observation?

Solution:

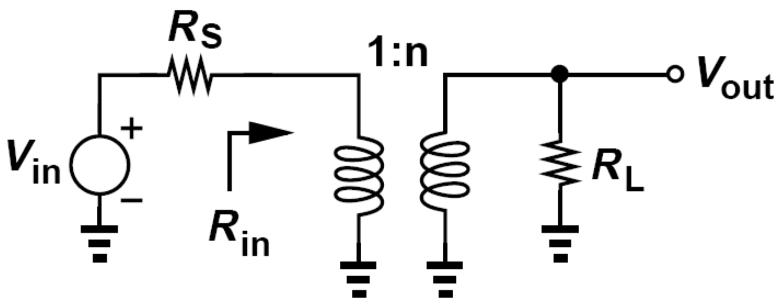


Yes, it is. Consider the arrangement shown above (left), where the passive network transforms R_L by a factor of α. Assuming the input port exhibits no imaginary component, we equate the power delivered to the network to the power delivered to the load:

$$\left(V_{in}\frac{\alpha R_L}{\alpha R_L + R_S}\right)^2 \cdot \frac{1}{\alpha R_L} = \frac{V_{out}^2}{R_L} \Rightarrow V_{out} = \frac{V_{in}}{\sqrt{\alpha}} \cdot \frac{R_L}{R_L + \frac{R_S}{\alpha}}$$

If the input and output ports of such a network are swapped, the resistance transformation ratio is simply inverted.

Impedance Matching by Transformers



$$V_{in}^2/R_{in} = n^2V_{in}^2/R_L$$

$$R_{in} = R_L/n^2$$

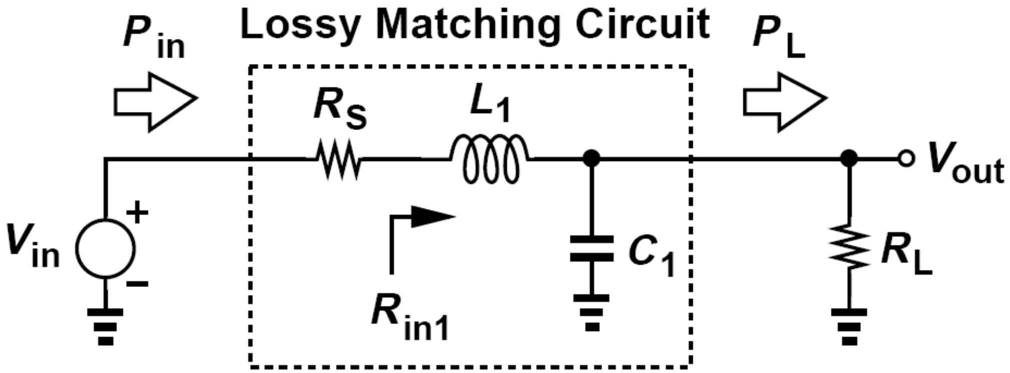
Loss in Matching Networks

We define the loss as the power provided by the input divided by that delivered to R_L

$$P_{in} = \frac{V_{in}^2}{R_S + R_{in1}}$$

$$P_L = \left(V_{in}\frac{R_{in1}}{R_S + R_{in1}}\right)^2 \cdot \frac{1}{R_{in1}}$$

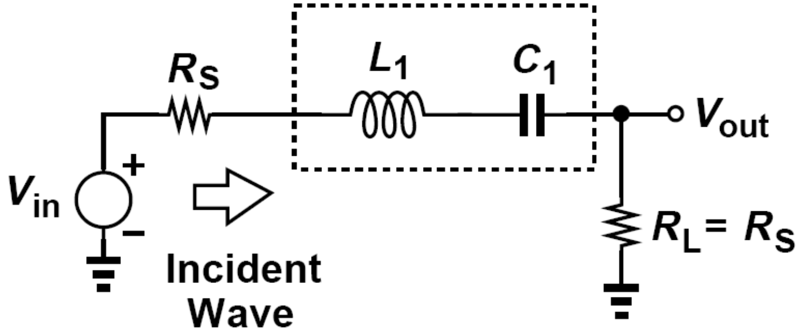
$$\begin{aligned}\text{Loss} &= \frac{P_{in}}{P_L} \\ &= 1 + \frac{R_S}{R_{in1}}\end{aligned}$$



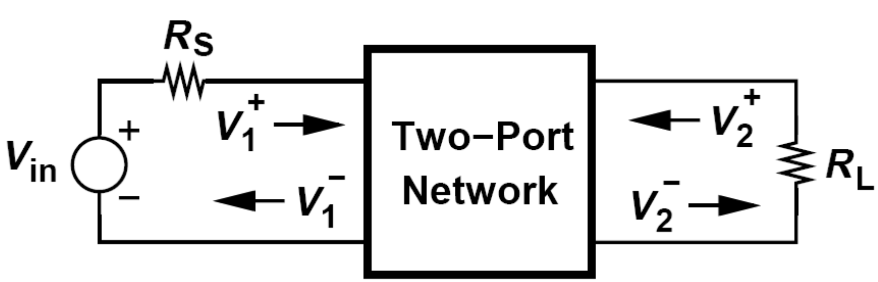
$$\begin{aligned}P_{in} &= \frac{V_{out}^2}{R_P || R_L} \\ &= \frac{V_{out}^2}{R_L}\frac{R_P + R_L}{R_P}\end{aligned}$$

$$\text{Loss} = 1 + \frac{R_L}{R_P}$$

Scattering Parameters

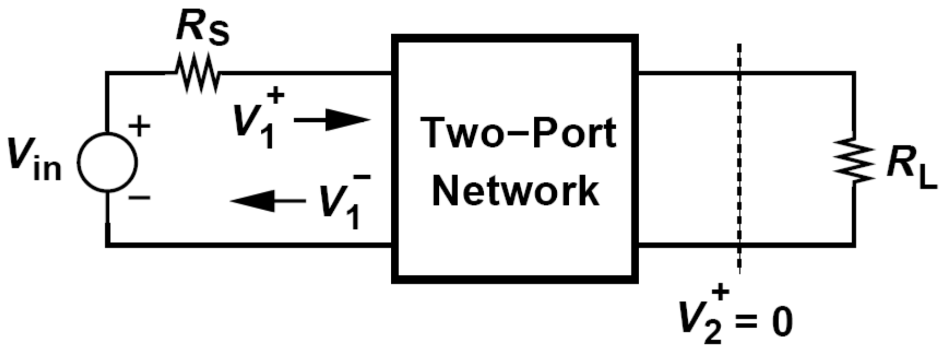


- **S-Parameter: Use power quantities instead of voltage or current**
- **The difference between the incident power (the power that would be delivered to a matched load) and the reflected power represents the power delivered to the circuit.**



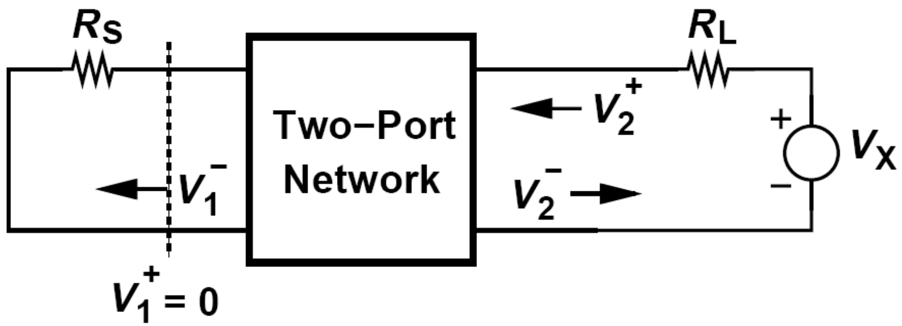
$$
\begin{aligned}
V_1^- &= S_{11}V_1^+ + S_{12}V_2^+ \\
V_2^- &= S_{21}V_1^+ + S_{22}V_2^+.
\end{aligned}
$$

S_{11} and S_{12}



$$S_{11} = \frac{V_1^-}{V_1^+}|_{V_2^+=0}$$

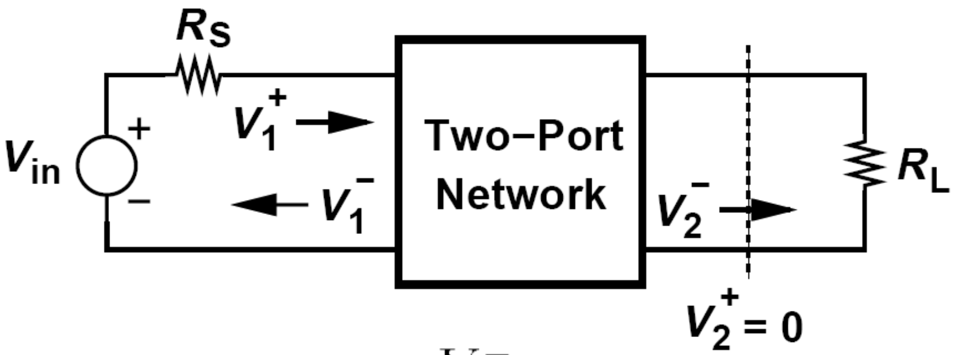
- S_{11} **is the ratio of the reflected and incident waves at the input port when the reflection from** R_L **is zero.**
- **Represents the accuracy of the input matching**



$$S_{12} = \frac{V_1^-}{V_2^+}|_{V_1^+=0}$$

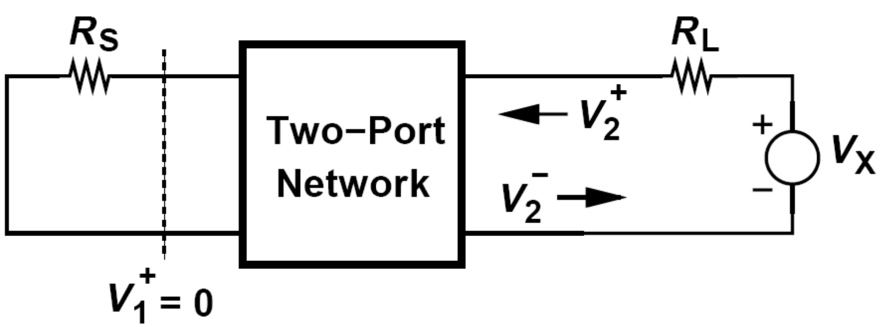
- S_{12} **is the ratio of the reflected wave at the input port to the incident wave into the output port when the input is matched**
- **Characterizes the *reverse isolation***

S_{21} and S_{22}



$$S_{21} = \frac{V_2^-}{V_1^+}\bigg|_{V_2^+=0}$$

- S_{21} **is the ratio of the wave incident on the load to that going to the input when the reflection from** R_L **is zero**
- **Represents the gain of the circuit**



$$S_{22} = \frac{V_2^-}{V_2^+}\bigg|_{V_1^+=0}$$

- S_{22} **is the ratio of reflected and incident waves at the output when the reflection from** R_s **is zero**
- **Represents the accuracy of the output matching**

Scattering Parameters: A few remarks

$$\begin{aligned} V_1^- &= S_{11}V_1^+ + S_{12}V_2^+ \\ V_2^- &= S_{21}V_1^+ + S_{22}V_2^+ \end{aligned}$$

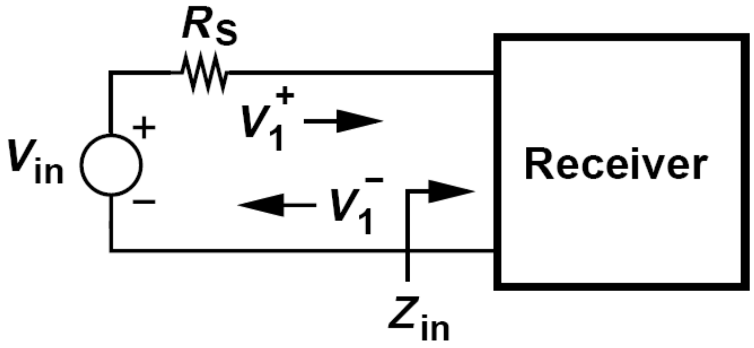
- **S-parameters generally have frequency-dependent complex values**
- **We often express S-parameters in units of dB**

$$S_{mn}|_{dB} = 20\log|S_{mn}|$$

- **The condition $V_2^+=0$ does not mean output port of the circuit must be conjugate-matched to R_L.**

Input Reflection Coefficient

In modern RF design, S_{11} is the most commonly-used S parameter as it quantifies the accuracy of impedance matching at the input of receivers.



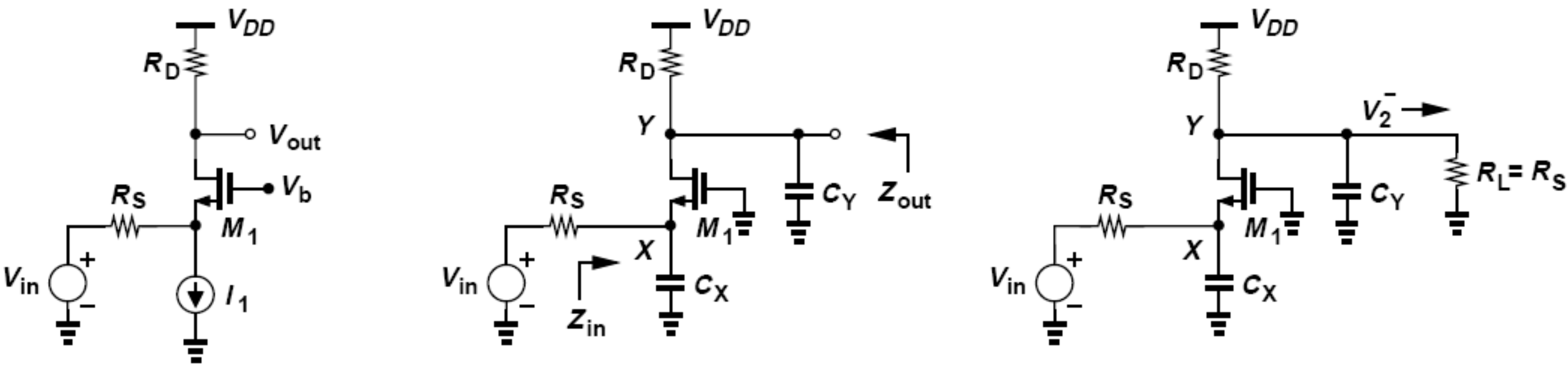
$$V_1^- = V_{in}\frac{Z_{in}}{Z_{in}+R_S} - \frac{V_{in}}{2} = \frac{Z_{in}-R_S}{2(Z_{in}+R_S)}V_{in}.$$

$$\frac{V_1^-}{V_1^+} = \frac{Z_{in}-R_S}{Z_{in}+R_S}.$$

- **Called the "input reflection coefficient" and denoted by G_{in}, this quantity can also be considered to be S_{11} if we remove the condition $V_2^+ = 0$**

Example of Scattering Parameters (Ⅰ)

Determine the S-parameters of the common-gate stage shown in figure below (left). Neglect channel-length modulation and body effect.



Drawing the circuit as shown above (middle), where $C_X = C_{GS} + C_{SB}$ and $C_Y = C_{GD} + C_{DB}$, we write $Z_{in} = (1/g_m)||(C_X s)^{-1}$ and

$$S_{11} = \frac{Z_{in} - R_S}{Z_{in} + R_S} = \frac{1 - g_m R_S - C_X s}{1 + g_m R_S + C_X s}$$

For S_{12}, we recognize that above arrangement yields no coupling from the output to the input if channel-length modulation is neglected. Thus, $S_{12} = 0$.

Example of Scattering Parameters (Ⅱ)

For S_{22}, we note that $Z_{out} = R_D||(C_Y s)^{-1}$ and hence

$$
\begin{aligned}
S_{22} &= \frac{Z_{out} - R_S}{Z_{out} + R_S} \\
&= -\frac{R_S - R_D + R_S R_D C_Y s}{R_S + R_D + R_S R_D C_Y s}
\end{aligned}
$$

Lastly, S_{21} is obtained according to the configuration of figure above (right). Since $V_2^-/Vin = (V_2^-/V_X)(V_X/V_{in})$, $V_2^-/V_X = g_m[R_D||R_S||(C_Y s)^{-1}]$, and $V_X/V_{in} = Z_{in}/(Z_{in} + R_S)$, we obtain

$$
\frac{V_2^-}{V_{in}} = g_m \left(R_D||R_S||\frac{1}{C_Y s} \right) \frac{1}{1 + g_m R_S + R_S C_X s}
$$

$$
S_{21} = 2g_m \left(R_D||R_S||\frac{1}{C_Y s} \right) \frac{1}{1 + g_m R_S + R_S C_X s}.
$$